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> #Title: RollesTheorem
#Author: Gord Clement, May 2011
#Description: For a given function and interval, this procedure determines if the conditions

#           of the mean Rolle's theorem hold. If so, all possible values  $c$  on the interval
which satisfy the conclusion

#           of the theorem are found. An animation shows the function plotted on the
interval with the tangent line
#           moving across the function, highlighting the  $c$  values when they are reached
#Usage:
#Call : RollesTheorem( function, interval)
#function : function to be used for the theorem, note : must be continuous on the given interval  $[a,$ 
 $b]$  and differentiable on  $(a, b)$  and  $f(a) = f(b)$ 

#interval: interval to be used in the theorem, entered in standard Maple notation, ie  $[a,b]$ 
would be entered  $x=a..b$ 
#For fully commented code, see MVT as logic is the same
RollesTheorem := proc( expr, range)
    local dir, clist, fullList, temp, i, j, found, output, var, a, b, step, tanslope, tempx, tany, tanline,
        aplot, bplot, secplot, funcplot, fAta, fAtb, tanplot, fullplot, miny, maxy, tempc, multiplec,
        previousStop, nextStop, length, maxdir, ctanplot;

    var := op( 1, range);
    a := evalf( op(1, op(2, range) ) );
    b := evalf( op(2, op(2, range) ) );
    miny := minimize( expr, var = a .. b );
    maxy := maximize( expr, var = a .. b );

    length := evalf(  $\frac{(b - a)}{5}$  );

    fAta := subs( var = a, expr );
    fAtb := subs( var = b, expr );

    dir := diff( expr, var );
    maxdir := maximize( |dir|, var = a .. b );
    found := false;
    output := -1;
    fullList := evalf( solve( dir = 0, var, dropmultiplicity = true ) );
    multiplec := false;
    if ( fAta  $\neq$  fAtb ) then
        output :=
            "Error: This function does not satisfy the conditions of Rolle's Theorem on the given range as  $f(a)$  does not
    elif (  $a \geq b$  ) then
        output := "Error: a must be less than b";
    elif ( is( type( maxdir, infinity ) ) ) then
        output
            := "Error: Function does not satisfy the conditions of the Mean Value Theorem since it is
            not differentiable on (a,b)";
    elif ( not( iscont( expr, var = a .. b, 'closed' ) ) ) then
        output
            := "Error: Function does not satisfy the conditions of the Rolle's Theorem on the given
            range";

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elif (dir = 0) then
    output
    := "Error: Do not use a constant function, as all values between a and b satisfy the Rolle's
    Theorem";
else
    if (nops( [fullList] ) = 1) then
        if (is( fullList < b ) and is( fullList > a )) then
            clist := fullList;
        else
            output
            := "Error: Function does not satisfy the conditions of the Rolle's Theorem on the given
            range";
            end if;
        else
            multiplec := true;
            clist := [ ];
            for i from 1 to nops( [fullList] ) do
                temp := fullList[i];
                if (is( temp < b ) and is( temp > a )) then
                    found := true;
                    clist := [op( clist ), temp];
                end if;
            end do;
            if (not( found )) then
                output
                := "Error: Function does not satisfy the conditions of the Rolle's Theorem on the given
                range";
                end if;

            end if;
        end if;
        if (not ( output == -1 )) then
            output;
        else

            if (not( multiplec )) then
                step := evalf(  $\left( \frac{(\textit{clist} - a)}{25} \right)$  );
                output := [ ];
                tempx := a;
                aplot := plot( [a, t, t = miny - 0.2 .. maxy + 0.2 ], linestyle = dash, color = black ) :
                bplot := plot( [b, t, t = miny - 0.2 .. maxy + 0.2 ], linestyle = dash, color = black ) :
                funcplot := plot( [var, expr, var = a - 0.2 .. b + 0.2 ], thickness = 2, color = black ) :
                secplot := plot( [t, slope · (t - a) + fAta, t = a .. b ], thickness = 2, color = red ) :

                for i from 1 to 25 do
                    tempx := tempx + step;
                    tanslope := evalf( subs( var = tempx, dir ) );
                    tany := evalf( subs( var = tempx, expr ) );
                    tanline := tanslope · (t - tempx) + tany;
                    tanplot := plot( [t, tanline, t = tempx - length .. tempx + length ], color = black,
                    thickness = 2 ) :
                    fullplot := plots[display]( [aplot, bplot, tanplot, funcplot, secplot ], title

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= typeset(var, "=", evalf(tempx)), labels = ["", ""]) :
    output := [op(output), fullplot] :
    end do;

    tanslope := evalf(subs(var = clist, dir));
    tany := evalf(subs(var = clist, expr));
    tanline := tanslope · (t - clist) + tany;
    ctanplot := plot([t, tanline, t = clist - length..clist + length], color = red, thickness
= 2) :
    fullplot := plots[display]([aplot, bplot, ctanplot, funcplot, secplot], title
= typeset("c=", clist), labels = ["", ""]) :
    for i from 1 to 10 do
        output := [op(output), fullplot] :
    end do;

    step := evalf( $\frac{(b - clist)}{25}$ );
    tempx := clist;
    for i from 1 to 25 do
        tempx := tempx + step;
        tanslope := evalf(subs(var = tempx, dir));
        tany := evalf(subs(var = tempx, expr));
        tanline := tanslope · (t - tempx) + tany;
        tanplot := plot([t, tanline, t = tempx - length..tempx + length], color = black,
thickness = 2) :
        fullplot := plots[display]([aplot, bplot, tanplot, funcplot, secplot, ctanplot], title
= typeset(var, "=", evalf(tempx)), labels = ["", ""]) :
        output := [op(output), fullplot] :
    end do;
    fullplot := plots[display]([aplot, bplot, funcplot, secplot, ctanplot], title
= typeset(var, "=", evalf(tempx)), labels = ["", ""]) :
    output := [op(output), fullplot] :
    plots[display]([op(output)], insequence = true, view = [a - 0.2 .. b + 0.2, miny - 0.2
..maxy + 0.2]);
else
    clist := sort(clist);
    aplot := plot([a, t, t = miny - 0.2 ..maxy + 0.2], linestyle = dash, color = black) :
    bplot := plot([b, t, t = miny - 0.2 ..maxy + 0.2], linestyle = dash, color = black) :
    funcplot := plot([var, expr, var = a - 0.2 .. b + 0.2], thickness = 2, color = black) :
    secplot := plot([t, slope · (t - a) + fAta, t = a .. b], thickness = 2, color = red) :
    output := [ ];
    tempx := a;
    previousStop := a;
    nextStop := evalf(clist[1]);
    ctanplot := [ ];
    for i from 1 to nops(clist) do
        tempc := evalf(clist[i]);
        step := evalf( $\frac{nextStop - previousStop}{25}$ );
    end do;

    for j from 1 to 25 do

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tempx := tempx + step;
tanslope := evalf(subs(var = tempx, dir));
tany := evalf(subs(var = tempx, expr));
tanline := tanslope * (t - tempx) + tany;
tanplot := plot([t, tanline, t = tempx - length..tempx + length], color
= black, thickness = 2) :
fullplot := plots[display]([aplot, bplot, tanplot, funcplot, secplot,
op(ctanplot)], title = typeset(var, "=", evalf(tempx)), labels = ["", ""]) :
output := [op(output), fullplot] :
end do;

tanslope := evalf(subs(var = tempc, dir));
tany := evalf(subs(var = tempc, expr));
tanline := tanslope * (t - tempc) + tany;
ctanplot := [op(ctanplot), plot([t, tanline, t = tempc - length..tempc
+ length], color = red, thickness = 2)]:
fullplot := plots[display]([aplot, bplot, tanplot, funcplot, secplot,
op(ctanplot)], title = typeset("c=", tempc), labels = ["", ""]) :
for j from 1 to 10 do
output := [op(output), fullplot] :
end do;
if (not(i = nops(clist))) then
previousStop := nextStop;
nextStop := evalf(clist[i + 1]);
else
previousStop := nextStop;
nextStop := b;
end if;
end do;
step := evalf( (nextStop - previousStop) / 25 );

for j from 1 to 25 do
tempx := tempx + step;
tanslope := evalf(subs(var = tempx, dir));
tany := evalf(subs(var = tempx, expr));
tanline := tanslope * (t - tempx) + tany;
tanplot := plot([t, tanline, t = tempx - length..tempx + length], color = black,
thickness = 2) :
fullplot := plots[display]([aplot, bplot, tanplot, funcplot, secplot,
op(ctanplot)], title = typeset(var, "=", evalf(tempx)), labels = ["", ""]) :
output := [op(output), fullplot] :
end do;
fullplot := plots[display]([aplot, bplot, funcplot, secplot, op(ctanplot)], title
= typeset(var, "=", evalf(tempx)), labels = ["", ""]) :
output := [op(output), fullplot] :
plots[display]([op(output)], insequence = true, view = [a - 0.2 .. b + 0.2, miny - 0.2
.. maxy + 0.2]);
end if;
end if;
end proc;

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> RollesTheorem( $x^2 - 4x + 1, x = 0..4$ )

> *RollesTheorem*( $x^3 - 3x^2 + 2, x=0..2$ )  
"Error: This function does not satisfy the conditions of Rolle's Theorem on the given range as f(a) does not equal f(b)" (1)

> *RollesTheorem*( $(x - 2) \cdot (x - 1) \cdot (x) \cdot \left(x + \frac{1}{2}\right) \cdot (x + 3), x=-3..2$ )

> *RollesTheorem*(*surd*( $x^2, 3$ ),  $x=-1..1$ )  
"Error: Function does not satisfy the conditions of the Mean Value Theorem since it is not differentiable on (a,b)" (2)

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