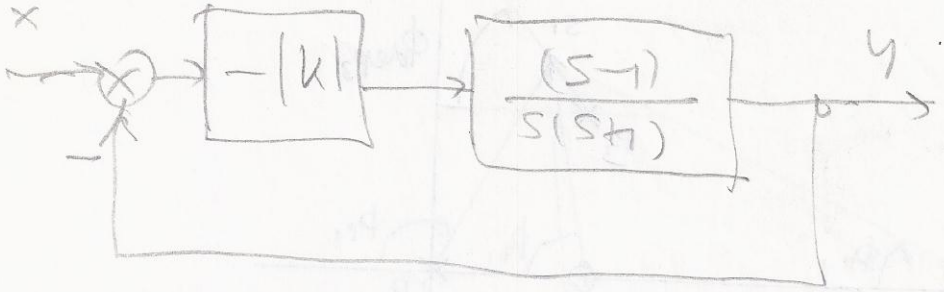


$$\frac{-|k|(s-1)}{s^2+s-|k|(s-1)} = \frac{k(1-s)}{s^2+(1-k)s+k}$$

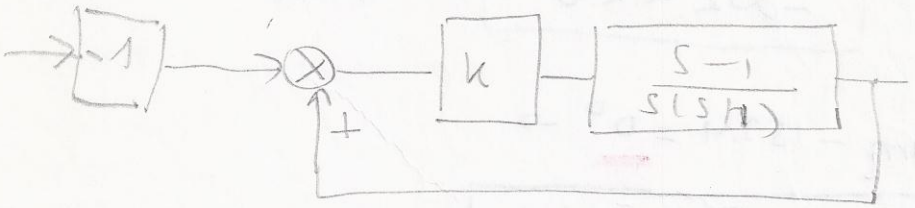
$k = 0.5$ ← [illegible]

$$\frac{y}{x} = \frac{0.5(1-s)}{s^2+0.5s+0.5}$$

$x = \frac{1}{s}$ [illegible]

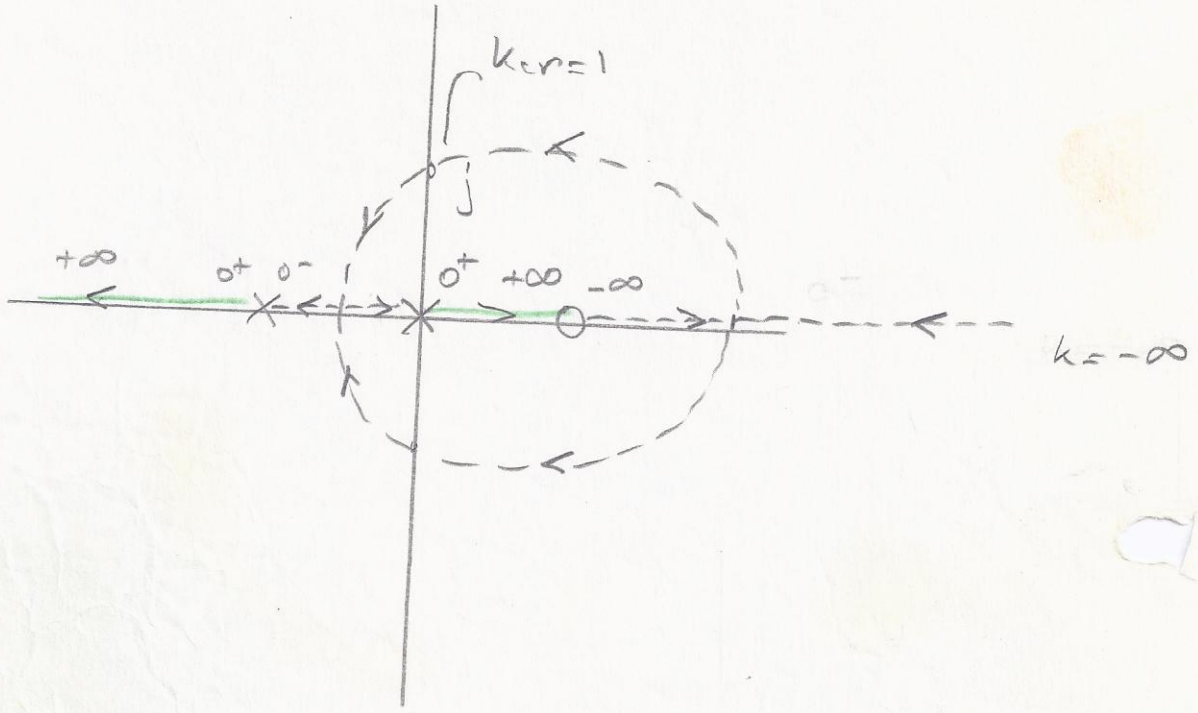
$$y_{ss} = \lim_{s\to 0} \frac{0.5(1-s)}{s^2+0.5s+0.5} = 1$$

Equivalent with:



$$\frac{G}{1-GH} = \frac{-1 \cdot k(s-1)}{s^2+s-k(s-1)}$$

$$G(s)H(s) = \frac{k(s-1)}{s(s+1)}$$



$$1 + GH = 0$$

$$k(s-1) + s(s+1) = 0 \rightarrow$$

$$s^2 + (k+1)s - k = 0$$

s^2	1	$-k$
s^1	$(k+1)$	0
s^0	$-k$	

$$s^1 \rightarrow (k+1) > 0 \rightarrow k > -1$$
$$s^0 \rightarrow k < 0$$

$$\boxed{-1 < k < 0}$$

$$s^2 - (-1) = 0 \rightarrow s_{1,2} = j$$

$$\frac{d}{ds}\left[\frac{1}{G(s)H(s)}\right]=0$$

$$\frac{d}{ds}\left[\frac{s(s+1)}{(s-1)}\right]=\frac{d}{ds}\left[\frac{s^2+s}{s-1}\right]=0 \rightarrow$$

$$(2s+1)(s-1)-s(s+1)=0$$

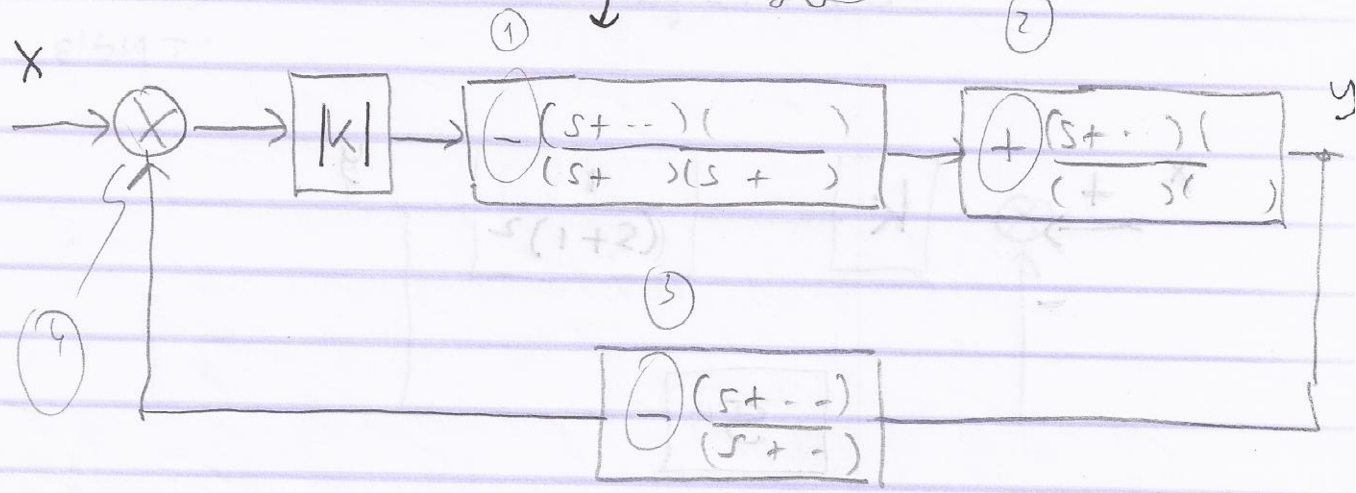
$$2s^2-2s+s-1-s^2-s=0$$

$$s^2-2s-1=0$$

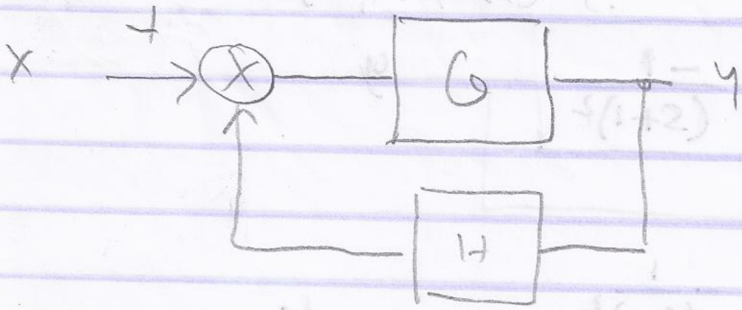
$$s_{1,2}=\frac{+2\pm\sqrt{4+4}}{2}=1\pm\sqrt{2}=\begin{matrix}-0.41,\\+2.41\end{matrix}$$

RL בורר

כללי:



$$\bar{G}_{c.c} = \frac{|K| \; \textcircled{1}(-) \; \textcircled{2}(+) \cdot \frac{(\;\;+\;\;)}{(\;\;+\;\;)}}{1 - \textcircled{4}(\times) |K| \; \textcircled{1}(-) \; \textcircled{2}(+) \; \textcircled{3}(-) \cdot \frac{(\;\;)(\;\;)}{(\;\;)(\;\;)}}$$



$$\bar{G} = \frac{G}{1 - \otimes \cdot \bigcirc \, GH}$$

↑ sign ↑ sign GH

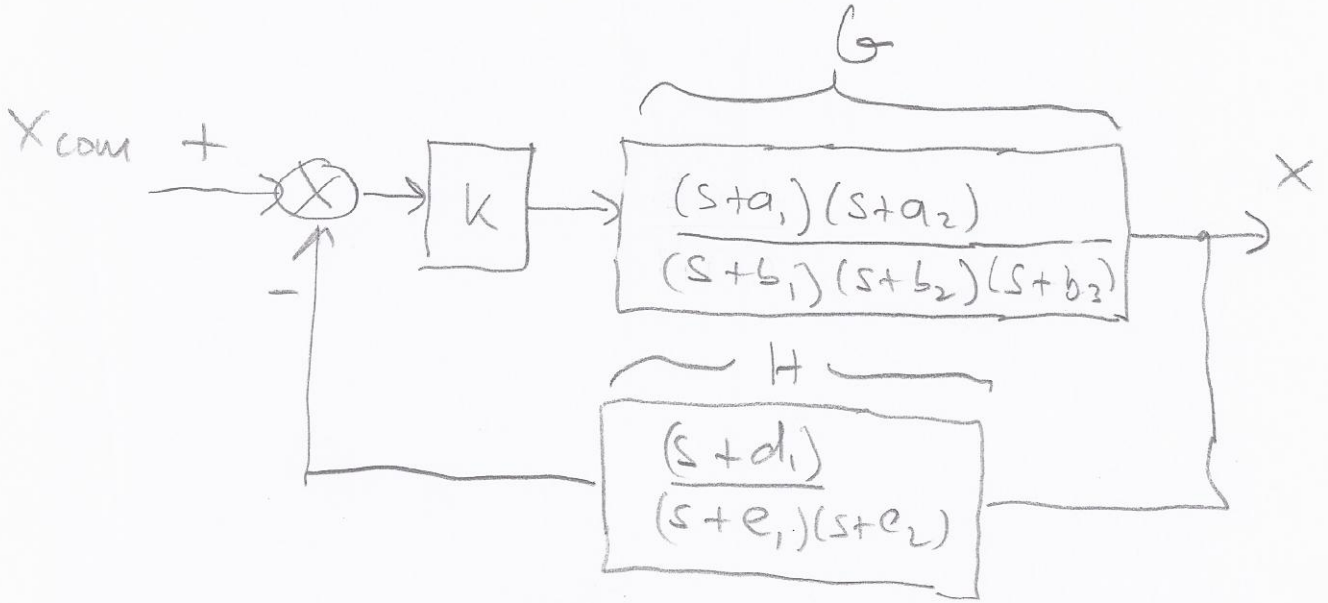
ZARL ← ϕ 0 +

RL ← $180°$ 0 −

$$K_{RL} \triangleq - \textcircled{4}(\times) |K| \; \textcircled{1} \; \textcircled{2} \; \textcircled{3}$$

↑ sign

אלגברה של מערכת בעלת מעגל סגור



$$\left.\frac{X}{X_{com}}\right|_{c.l} = \frac{kG}{1+kGH} \Rightarrow$$

$$= \frac{\frac{k\cdot(s+a_1)(s+a_2)}{(s+b_1)(s+b_2)(s+b_3)}}{1+\frac{k\cdot(s+a_1)(s+a_2)}{(s+b_1)(s+b_2)(s+b_3)}\cdot\frac{(s+d_1)}{(s+e_1)(s+e_2)}} =$$

$$= \frac{k\cdot(s+a_1)(s+a_2)(s+e_1)\cdot(s+e_2)}{\Delta(s)}$$

$\Delta(s)$ ← משוואה אופיינית בעלת מעגל סגור ⇐ RL

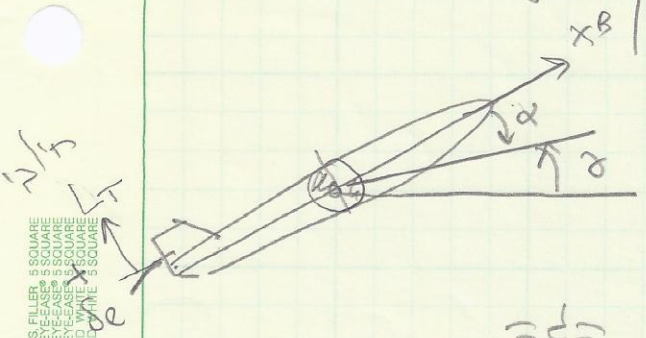
הגדרה: $N = RL$

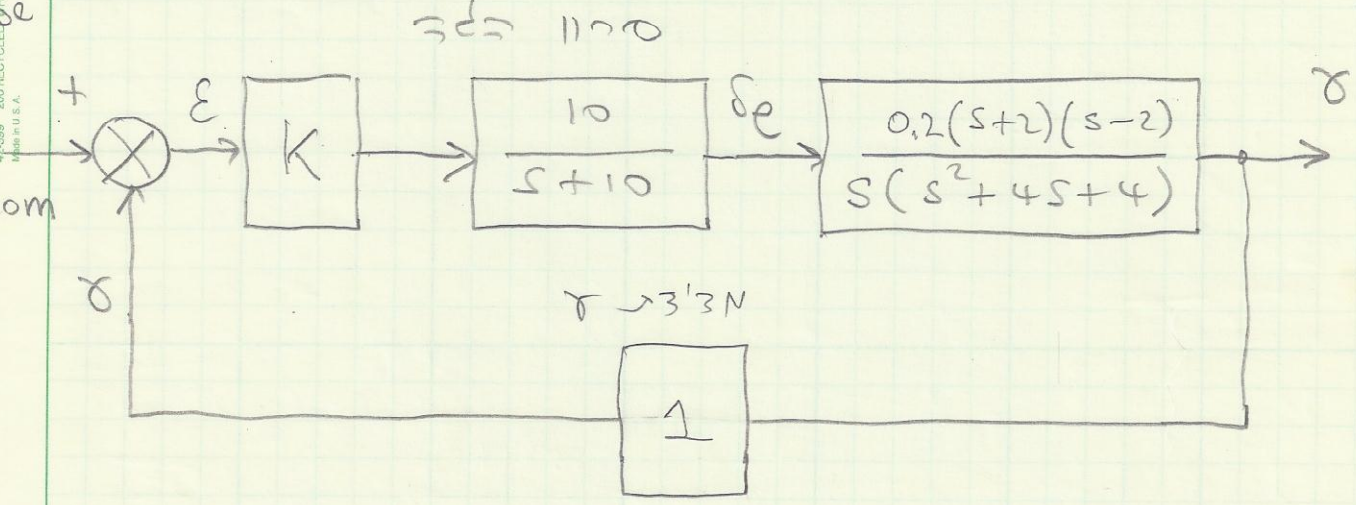
אלגברה

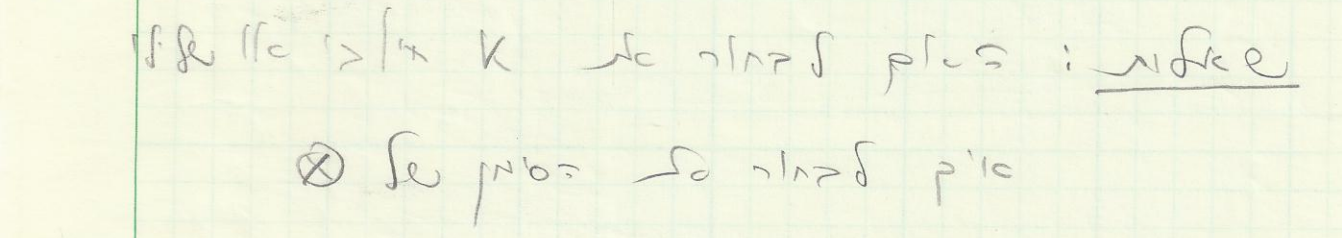
(1) אלגברה של G: נשארים

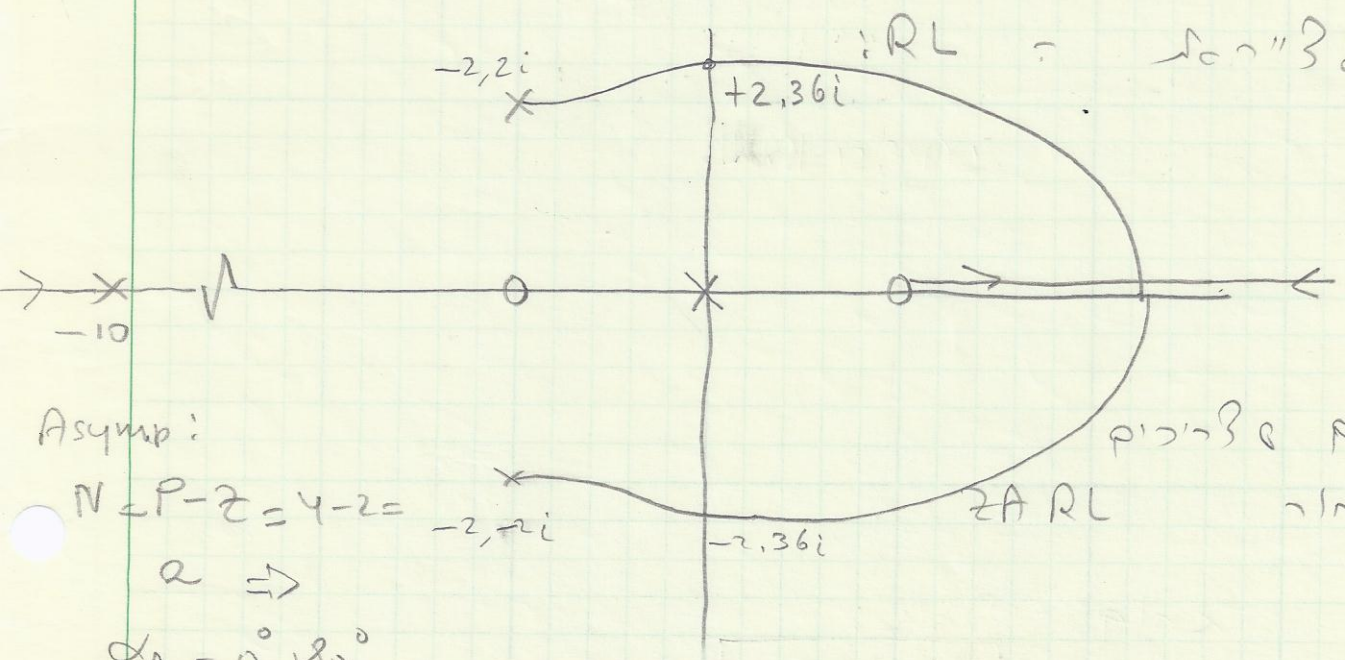
(2) אלגברה של H: נעלמים

(3) הגדרים של H: הופכים לחלק אלגברה בעלת מעגל סגור!


α
β
e

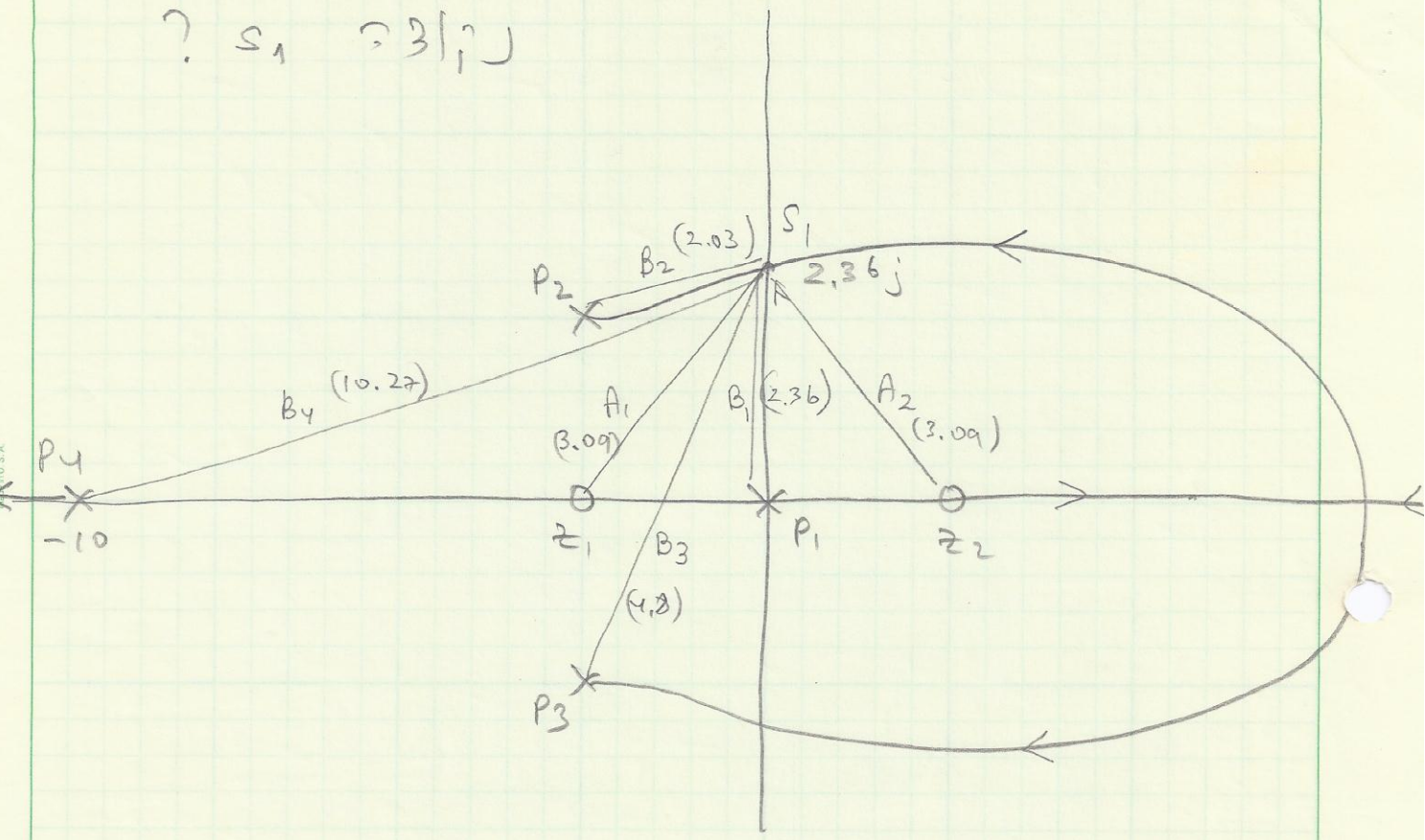

+
ε
K
10
s+10
e
0,2(2-s)(s+2)
s(s²+4s+4)
r
y
1




-2,2i
+2,36i
-10
-2,2i
-2,36i

PRL נתון כ-N

? S_1 מצא

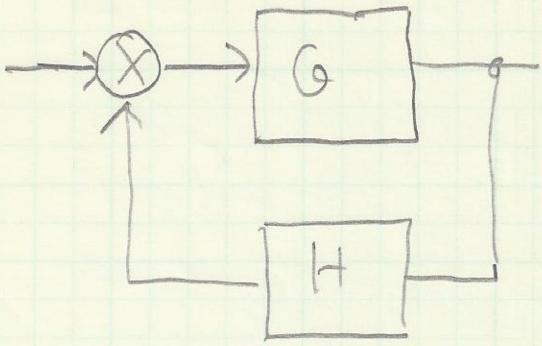


$P_1 = 0, 0i$	$P_1 - S_1 = 0, -2.36i$	$B_1 = 2,36$
$P_2 = -2, 2i$	$P_2 - S_1 = -2, -0.36i$	$B_2 = 2,03$
$P_3 = -2, -2i$	$P_3 - S_1 = -2, -4.36i$	$B_3 = 4.80$
$P_4 = -10, 0$	$P_4 - S_1 = -10, -2.36i$	$B_4 = 10,27$
$Z_1 = -2, 0$	$Z_1 - S_1 = -2, -2,36i$	$A_1 = 3,09$
$Z_2 = 2, 0$	$Z_2 - S_1 = 2, -2,36i$	$A_2 = 3.09$

$$|G_H| = \left| K_{RL} \cdot \frac{A_1 A_2}{B_1 B_2 B_3 B_4} \right| = 1 \qquad |K_{RL}| = \frac{2,36 \cdot 2,03 \cdot 4.80 \cdot 10,27}{(3,09)^2} = 24.7$$

בתנאי סף $|K_{RL}| = 24.7$

$-24.7 < K_{RL} < 0$



$$\bar{G} = \frac{G}{1+GH}$$

$$\bar{G} = \frac{G}{1 - \text{sign}\otimes \cdot GH}$$

$$\bar{G} = \frac{G}{1 - \text{sign}\otimes \underbrace{\left(K_{loop}\right)\frac{s^m(s+a)(\ \ )}{s^n(s+b)(\ \ )}}_{\triangleq GH}}$$

⇑

"צורת RL"

$$K_{RL} \triangleq -\text{sign}\otimes \cdot K_{loop}$$

RL 180 K_{RL} חיובי

ZARL K_{RL} שלילי

<u>אפשרות 1</u> k חיובי $\otimes$ חיובי ⇐

$$K_{RL} = -(\text{sign}\otimes)\, K_{loop} = -K_{loop} = -2k$$
$$\qquad\qquad + \qquad\quad +$$

לכן: מוצאים תחום יציבות

$$\boxed{-24.7 < K_{RL} < 0}$$

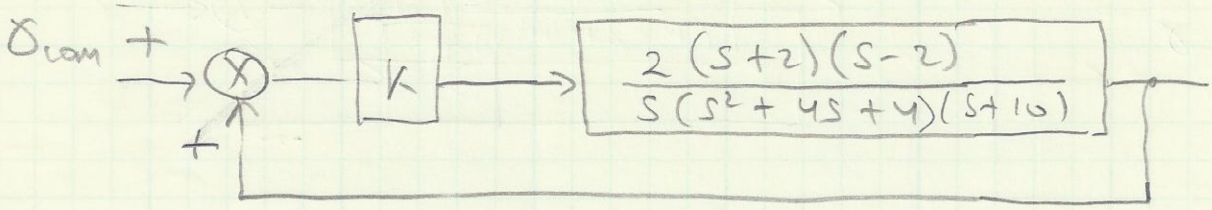
$$k = -\tfrac{1}{2} K_{RL}$$

$$\boxed{0 < k < 12.35}$$ לכן

K חיובי $\otimes$ חיובי

$$(K)\ (10)\ (0.2)\ \otimes = + \Rightarrow \text{ZARL}$$

$$\begin{matrix}\uparrow & \uparrow & \uparrow & \uparrow \\ + & + & + & +\end{matrix}$$



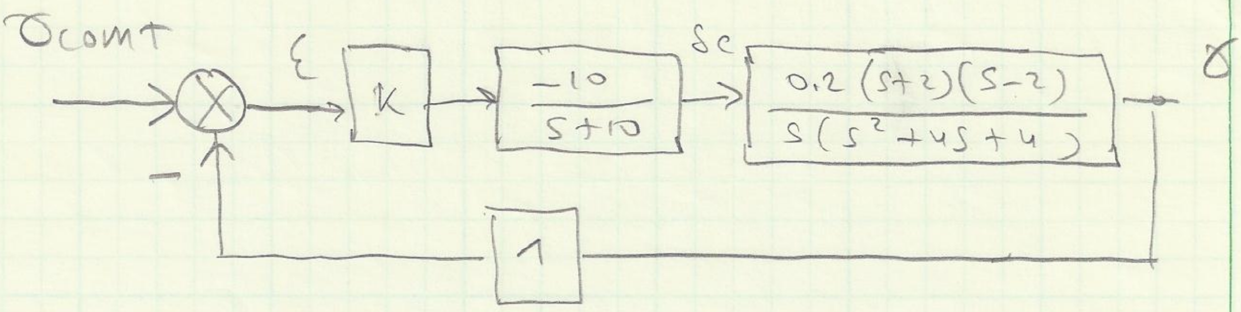
$$\bar{G} = \frac{2K \cdot \frac{(s^2-4)}{s(s^2+4s+4)(s+10)}}{1 - \frac{2K(s^2-4)}{s(s^2+4s+4)(s+10)}}$$

$$\bar{G} = \frac{2Ks^2 - 8K}{s^4 + 14s^3 + (44-2K)s^2 + 40s + 8K} = \frac{\gamma}{\gamma_{com}}$$

$$\gamma_{com} = \frac{\gamma_0}{s} \qquad \gamma_{ss} = \lim_{s\to 0} s\gamma(s) = -\gamma_0 \ !!$$

לכן $\boxed{\gamma_{ss} = -\gamma_0}$

לכן: צריכים להפוך את הסימן בחוג הקדמי



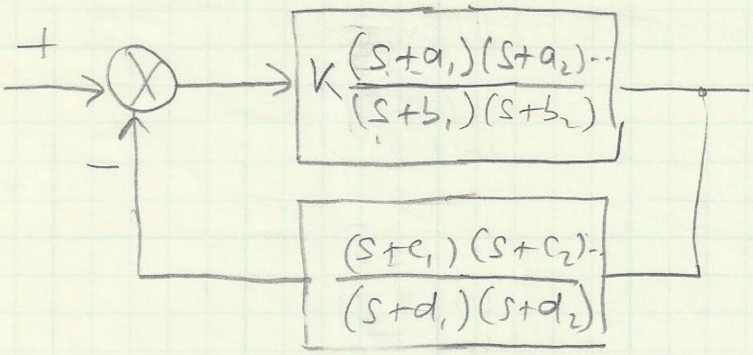
$$K_{RL} = -\mathrm{sig}\otimes \cdot K_{loop} = -\underbrace{\mathrm{sign}\otimes}_{-1} \cdot (-2k) = -2k$$

K חיובי K_{RL} שלילי ⇐ ZARL

$$\boxed{\gamma_{ss} = \gamma_0}$$

RL נותן רק : קטבים בחוג סגור

מה עם האפסים?



$$\bar{G} = \frac{G}{1+GH} = \frac{\dfrac{k(s+a_1)(s+a_2)\cdots}{(s+b_1)(s+b_2)}}{1+k\dfrac{(s+a_1)(s+a_2)(s+c_1)(s+c_2)}{(s+b_1)(s+b_2)(s+d_1)(s+d_2)}}$$

$$\bar{G} = \frac{K \cdot (s+a_1)(s+a_2)(s+d_1)(s+d_2)\cdots}{\Delta(s)}$$

לכן: (1) אפסים בחוג קדמי נשארים

(2) אפסים של המשוב נעלמים

(3) קטבים של המשוב הופכים להיות אפסים

Root Contours

הג) - קונטור (RC) $\triangleq$ מיקום הגאומטרי
של קטבי המערכת בחוג סגור כפונקציה
של פרמטר <u>כלשהו</u> (לאו דווקא הגבר החוג)

קטבי חוג סגור:
הפתרונות של: $1 + GH = 0$

עם $GH = K \cdot \frac{Z(s)}{P(s)}$

$$K \frac{Z(s)}{P(s)} + 1 = 0$$

$$K Z(s) + P(s) = 0$$

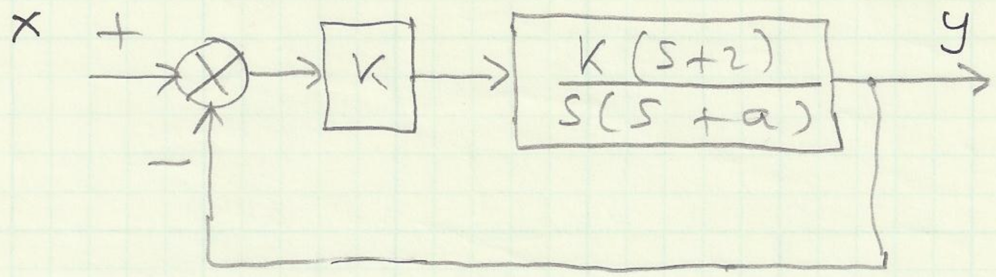
אם K קבוע, ורוצים את השורשים
כפונקציה של פרמטר כלשהו: τ
אפשר לבודד את τ:

$$\tau \tilde{Z}(s) + \tilde{P}(s) = 0$$

וניצור את ה- RC עבור $\tilde{G}\tilde{H} = \frac{\tilde{Z}(s)}{\tilde{P}(s)}$

בוחרים k קבוע, משנים של a :

דוגמא ה :



$$GH = \frac{k(s+2)}{s(s+a)}$$

$$1 + GH = 0$$

$$\frac{k(s+2)}{s(s+a)} + 1 = 0$$

$$ks + 2k + s^2 + as = 0 \Rightarrow$$

הפרמטר ← $as + (s^2 + ks + 2k) = 0$

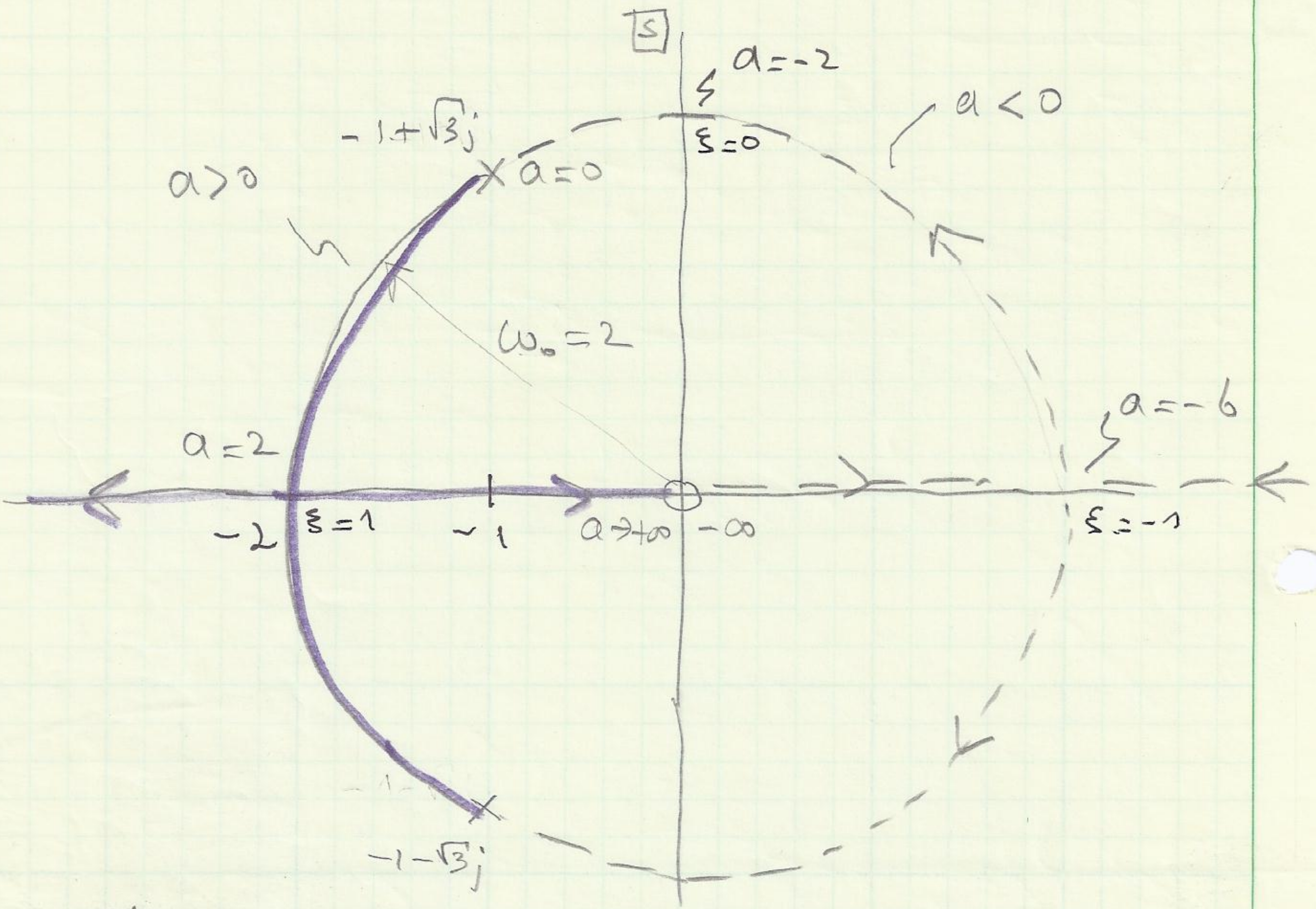
$$\frac{as}{s^2+ks+2k} + 1 = 0$$

$$\tilde{G}\tilde{H} = a\frac{\tilde{Z}(s)}{\tilde{P}(s)} = a\cdot\frac{s}{s^2+ks+2k}$$

$\boxed{k=2}$ נניח ש

$$\tilde{G}\tilde{H} = a\cdot\frac{s}{s^2+2s+4}$$

$$s_{1,2} = \frac{-2\pm\sqrt{4-16}}{2} = -1\pm\sqrt{3}j$$



$1+GH=0 \Rightarrow$

$\Delta(s) = as + s^2 + ks + 2k = 0$

$\Delta(s) = s^2 + \underbrace{(k+a)}_{2\zeta\omega_0} s + \underbrace{2k}_{\omega_0^2} = s^2 + (2+a)s + 4$ ($k=2$; $\Uparrow \omega_0^2$)

$\boxed{a=-2} \Leftarrow a+2=0 \Leftarrow$ חיתוך ציר $j\omega$

$\boxed{a_{1,2}=2,-6} \Leftarrow (a+2)=\pm 4$: קטבים כפולים ($b^2-4ac=0$)

$\zeta=1 \rightarrow$
$2\zeta\omega_0 = 2\cdot 2 = 4$

$2\zeta\omega_0 = 2\sqrt{2} = a+2 \Leftarrow \zeta = \frac{1}{2}\sqrt{2}$

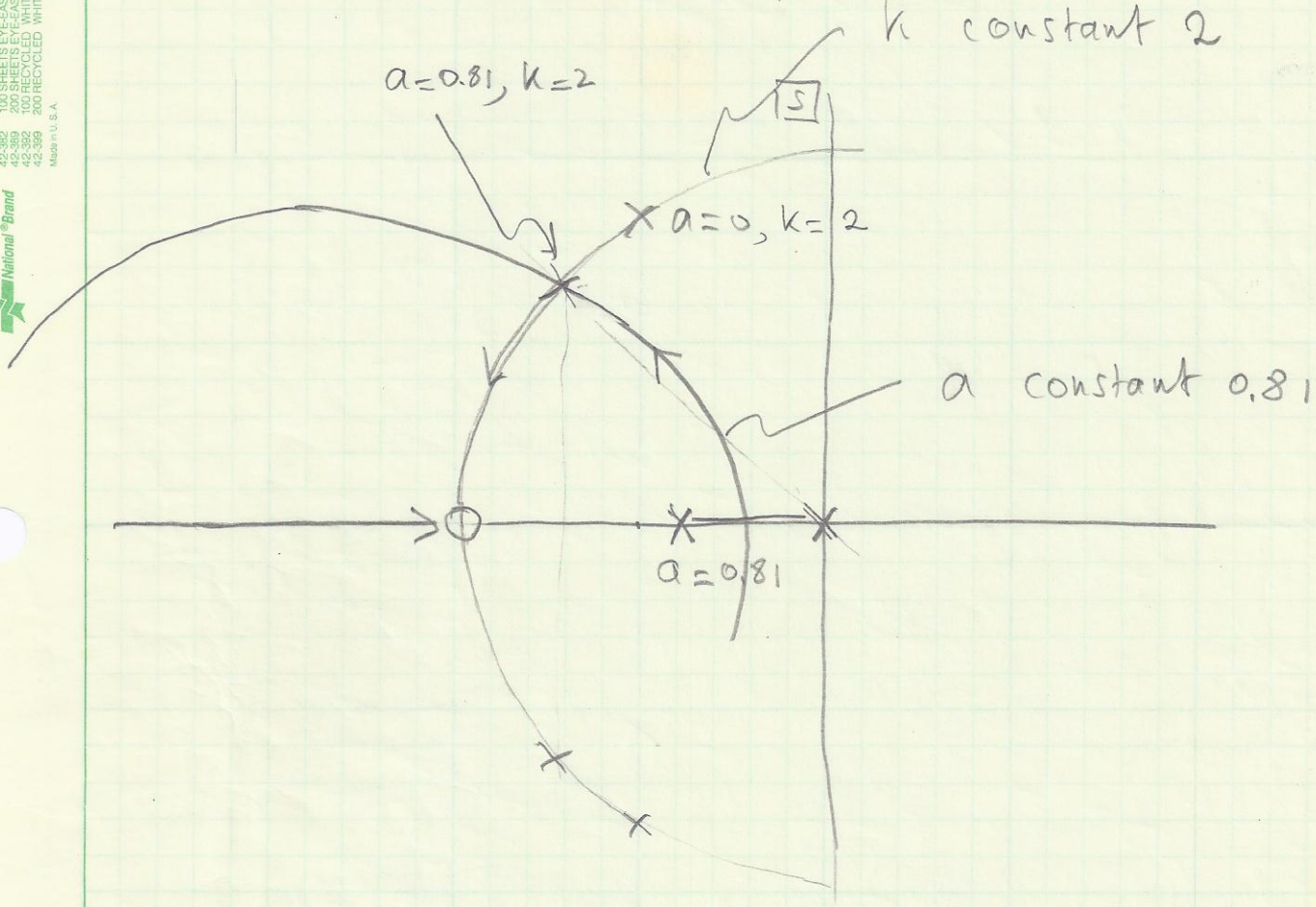
$a = 2\sqrt{2}-2 = 0.81$ [illegible]

$s^2 + 2\sqrt{2}s + 4 = 0$

$s_{1,2} = -\sqrt{2} \pm \sqrt{2}j$

: a = 0.81 ולפי RL

$$GH = \frac{k(s+2)}{s(s+0.81)}$$



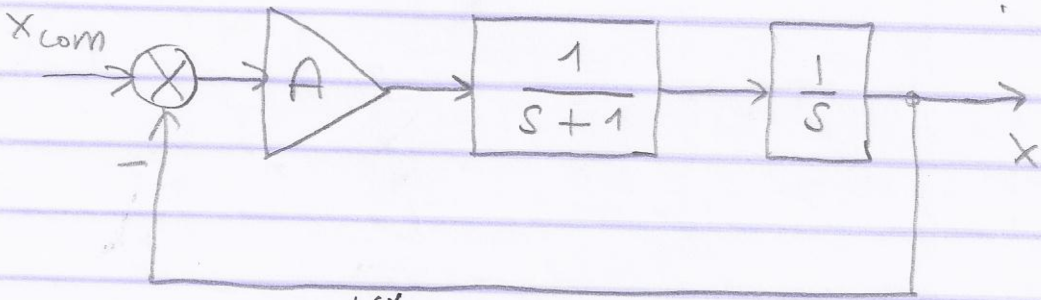
השאלות מקף:

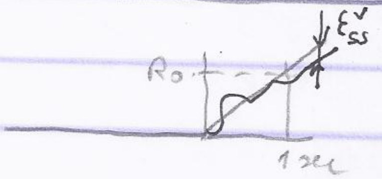
המטרה: לעמוד גם בדרישות סטטיות ודינמיות

1. wo
2. ריסון
3. פיגור

שיטת ה wo

הבעיה:





דרישות: (1) $\varepsilon_{ss}^{V} \leq 0.25\, R_0$

(2) $\zeta \geq 0.5$

(1) כדי לעמוד בדרישה הראשונה:

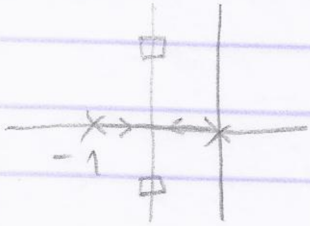
$$k_V \triangleq \lim_{s\to 0} s\,G(s) = A$$

$$\varepsilon_{ss}^{V} = \frac{R_0}{k_V} \leq 0.25\, R_0 \Rightarrow \boxed{k_V \geq 4}$$

$$\Rightarrow A \geq 4$$

(2) הדרישה השניה:

$$\Delta(s) = 1 + \frac{A}{s(s+1)} = 0 \Rightarrow$$



$$s^2 + s + A = 0$$ עבור $A=4$

$$\omega_0 = \sqrt{4} = 2 \quad ; \quad 2\zeta\omega_0 = 1 \Rightarrow \zeta = \frac{1}{2\omega_0} = 0.25$$

לא עומד בדרישה הדינמית

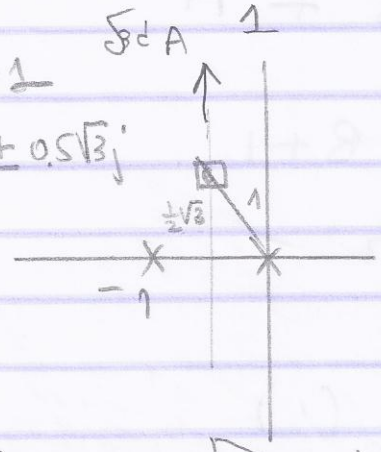
כדי לעמוד בדרישה הראשונה: $\zeta = 0.5$

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0$$

$$s^2 + \underbrace{2 \cdot 0.5 \cdot \omega_0}_{1} s + \underbrace{\omega_0^2}_{1} = 0 \qquad \omega_0^2 = 1 \Rightarrow A = 1$$

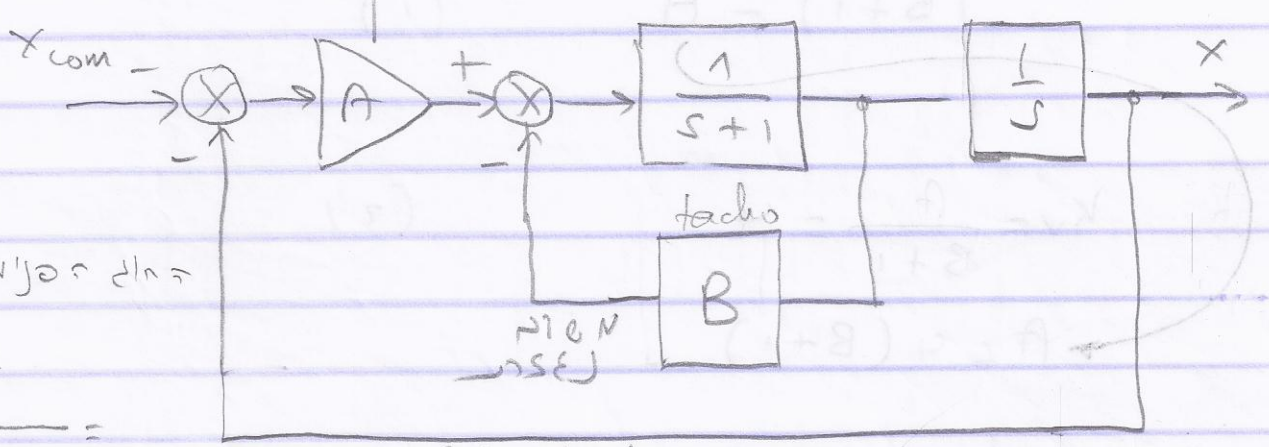
$$s^2 + s + 1$$

$$s_{1,2} = -0.5 \pm 0.5\sqrt{3}j$$



$$\Rightarrow K_v = 1 \qquad \underline{\xi_{ss} = R_0}$$

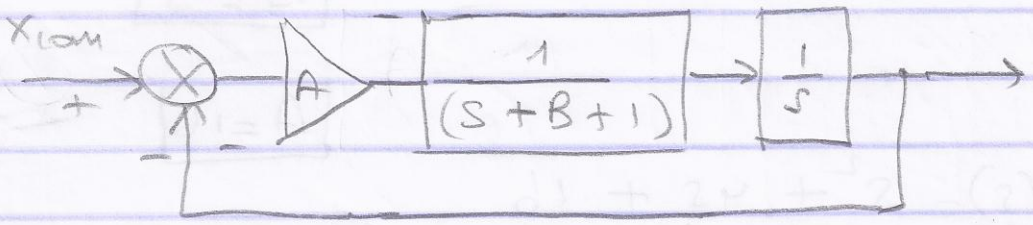
לרכיב משוב קיבולי:



פתרון הפונקציה:

$$\frac{\frac{1}{s+1}}{1 + \frac{B}{s+1}} = \frac{1}{s+1+B}$$

משוב מלא



$$\Delta(s) = 1 + \frac{A}{s(s+B+1)} = s^2 + (B+1)s + A$$

רווח נקי ניתן לשינוי

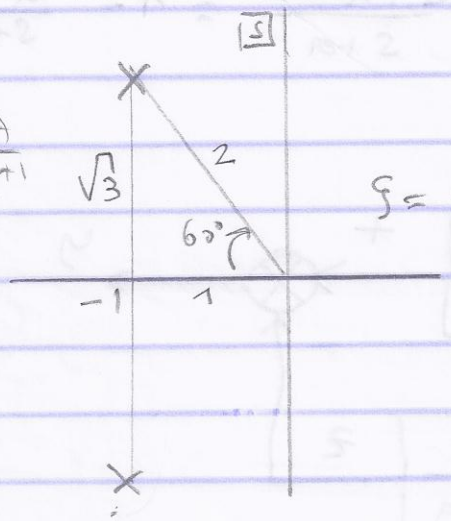
$$\boxed{A=4}$$

$$(B+1) = 2\zeta\omega_0 = 2 \cdot 0.5 \cdot 2 = 2$$

$$\boxed{B=1}$$

$$\Delta(s) = s^2 + 2s + 4$$

$$s_{1,2} = \frac{-2 \pm \sqrt{4-4\cdot 4}}{2} = -1 \pm j\sqrt{3}$$



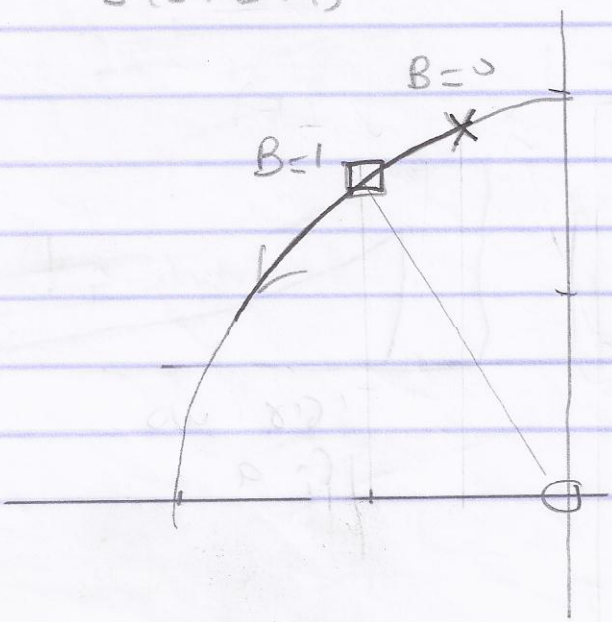
$$K_V = \lim_{s\to 0} s \cdot \frac{A}{s(s+B+1)} = \frac{A}{B+1}$$

$$K_V = \frac{4}{2} = 2$$

$$\zeta = \cos 60^\circ = 0.5$$

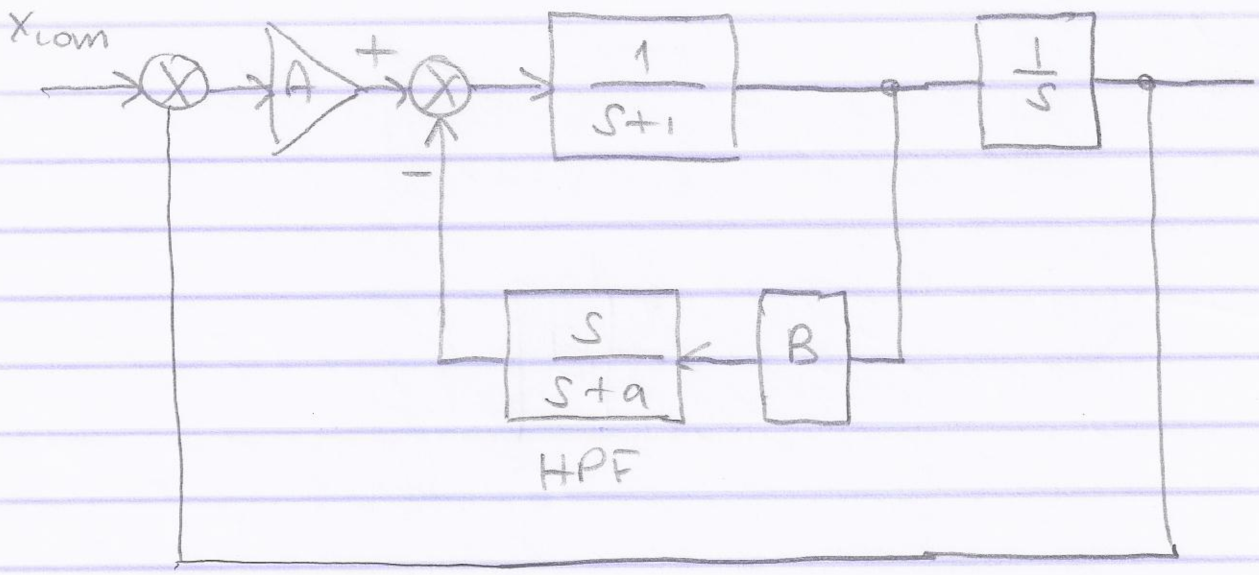
$$\varepsilon_{ss} = \frac{R_0}{2} = 0.5\,R_0$$

$$1 + \frac{A}{s(s+B+1)} = s^2 + s + A + Bs = 0$$



$$B \cdot \frac{s}{s^2+s+4}$$

W.O

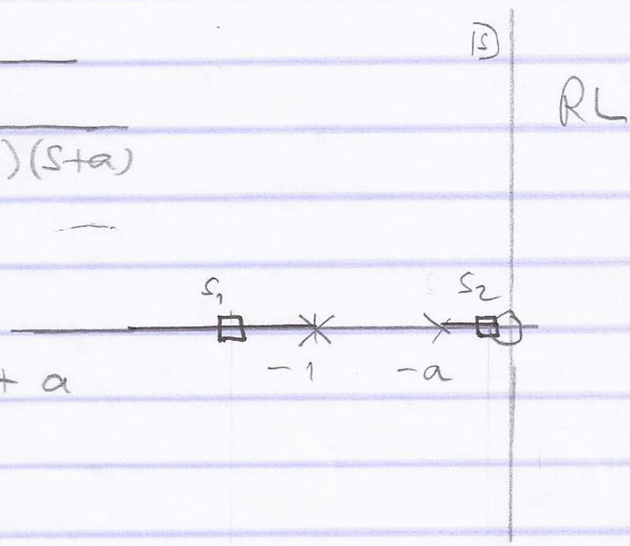


ה Wo הוא HPF שמעניין ב DC

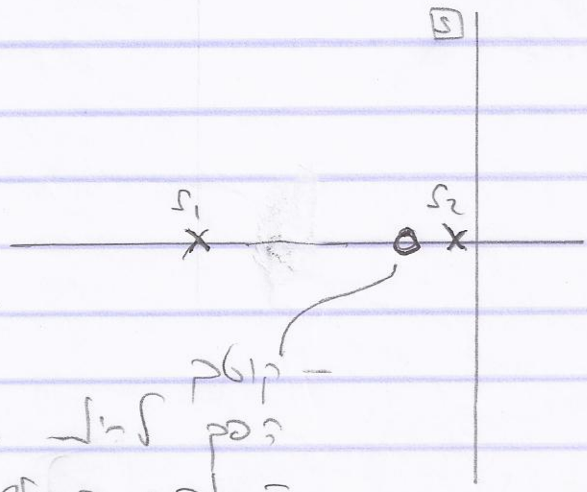
חוג הפנימי :

$$\bar{G} = \frac{\frac{1}{S+1}}{1 + B \cdot \frac{S}{(S+1)(S+a)}}$$

$$\bar{G} = \frac{S+a}{S^2 + (1+a+B)S + a}$$



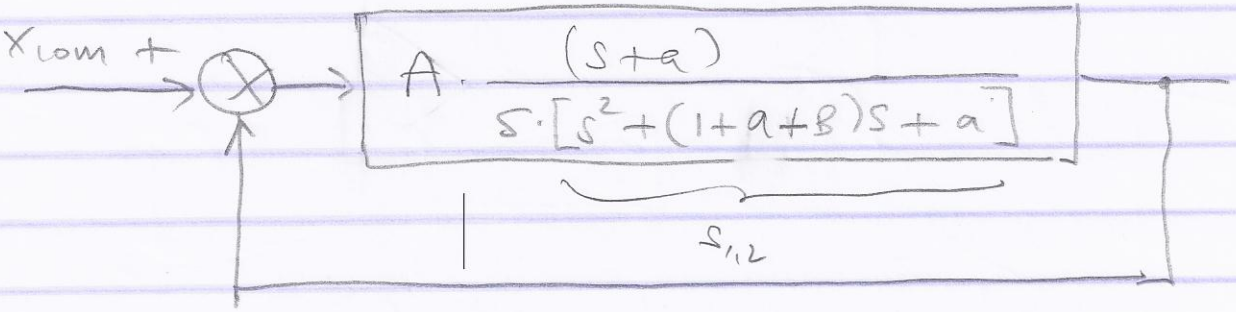
חוג סגור :



- קוטב הפך ל[illegible] אפס!

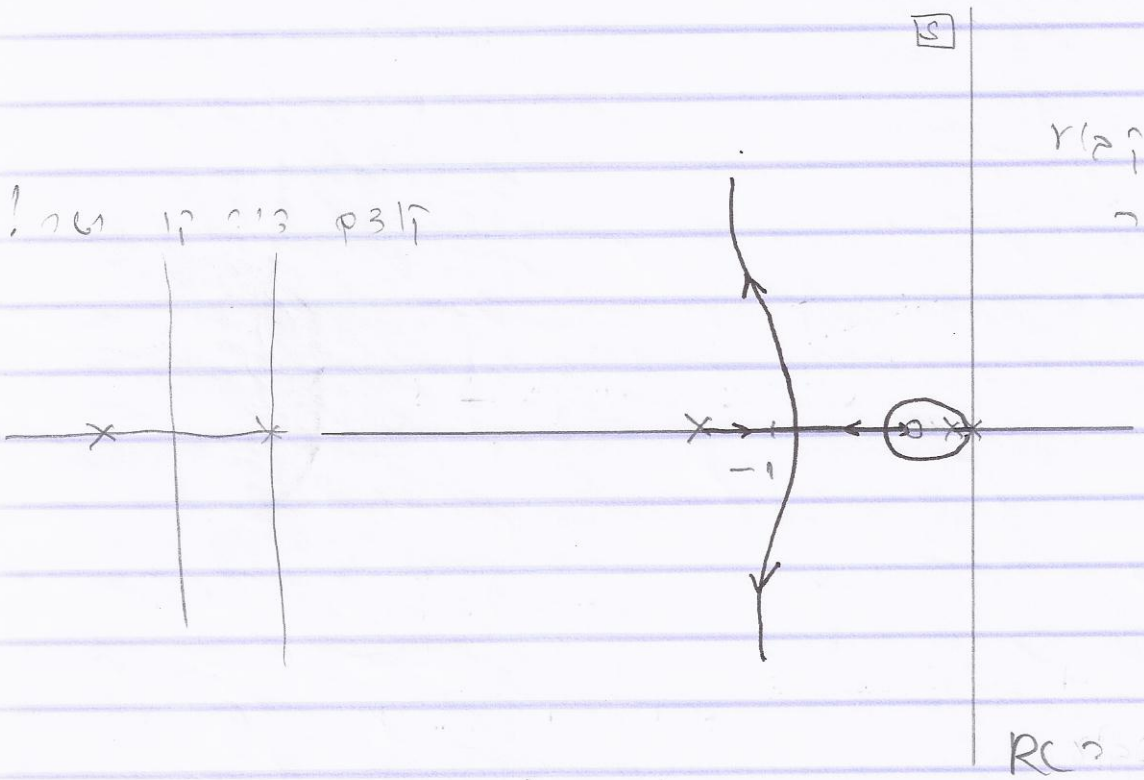
- האפס בקוטב נעלם

נתון המערכת:



כאן נלקח! $\Rightarrow$ $K_V = \lim_{s\to 0} sG(s) = A$

לכן: לדאוג ש $A=4$



נצייר כרגע את ה RL
עבור a
הכי גדול
שלנו נבחר:

$$\Delta(s) = 1 + A\frac{(s+a)}{s[s^2+(1+a+B)s+a]}$$

$$s^3 + (1+a+B)s^2 + (a+A)s + Aa = 0$$

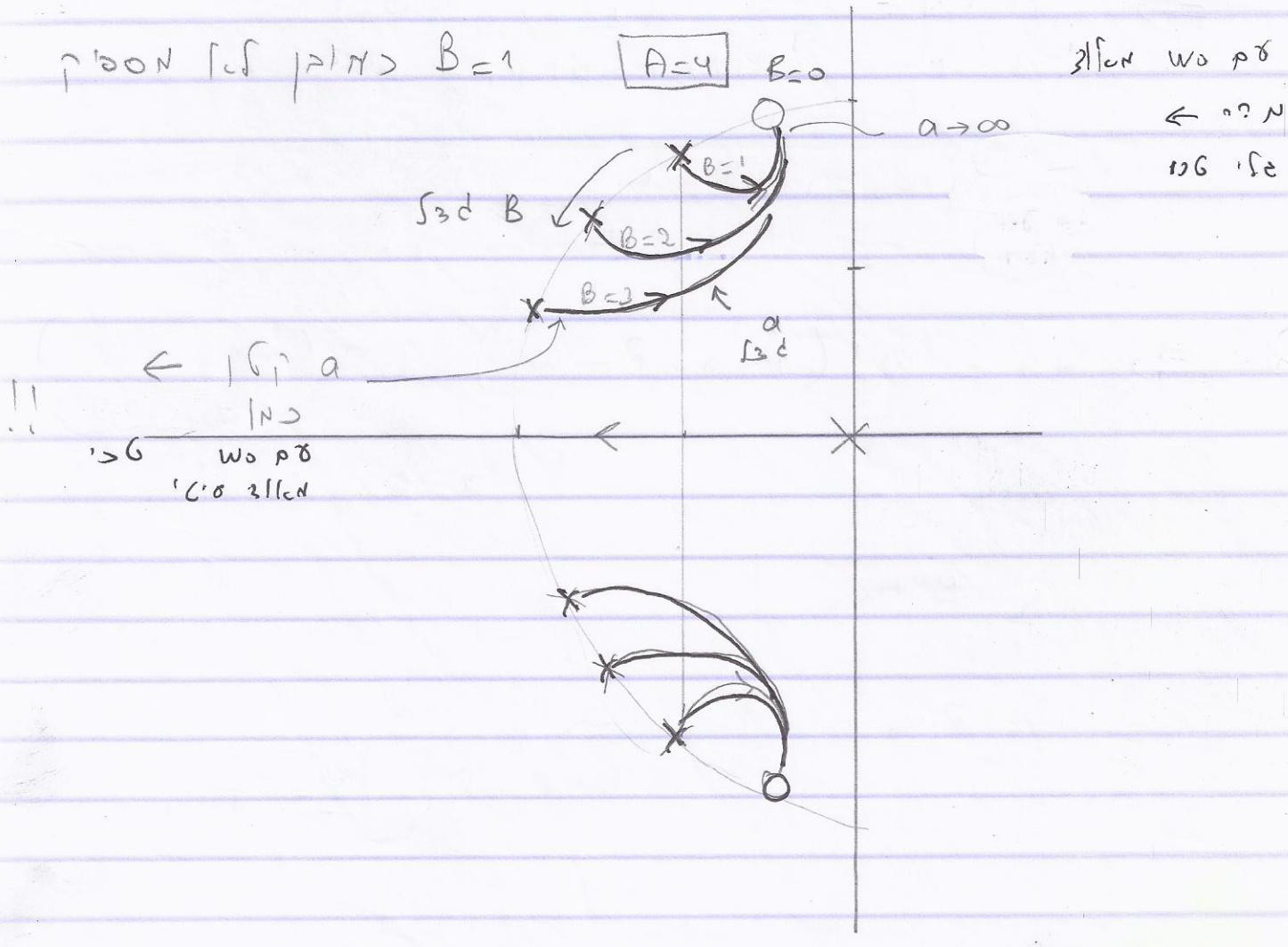
$$s^3 + (1+B)s^2 + As + as^2 + as + aA$$

$$s\left[s^2 + (1+B)s + A\right] + a\left[s^2 + s + A\right]$$

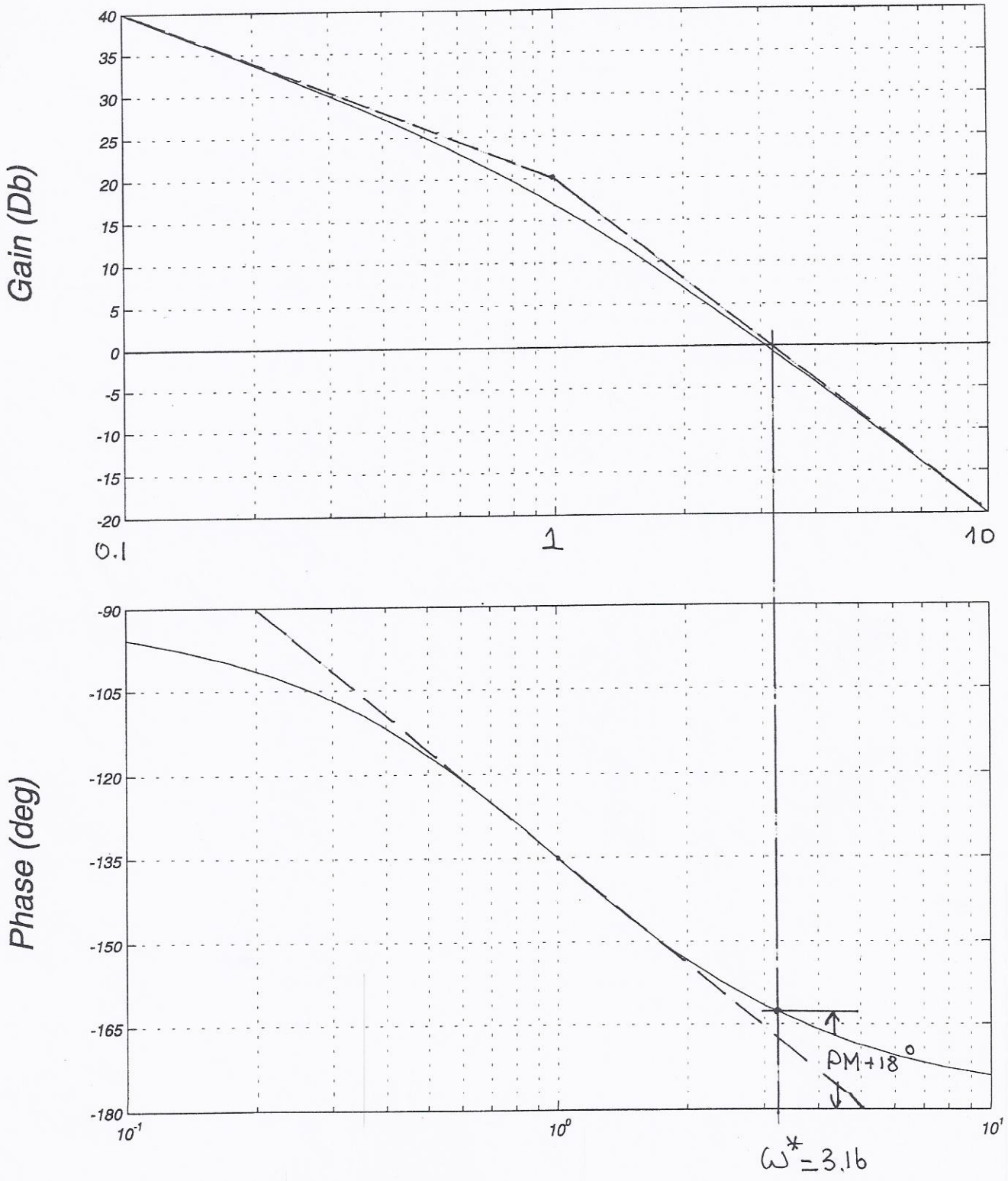
$$\widetilde{G}\widetilde{H} = a \cdot \frac{(s^2 + s + A)}{s\left[s^2 + (1+B)s + A\right]}$$

נציב: $A=4$, $B=1$ ונקבל:

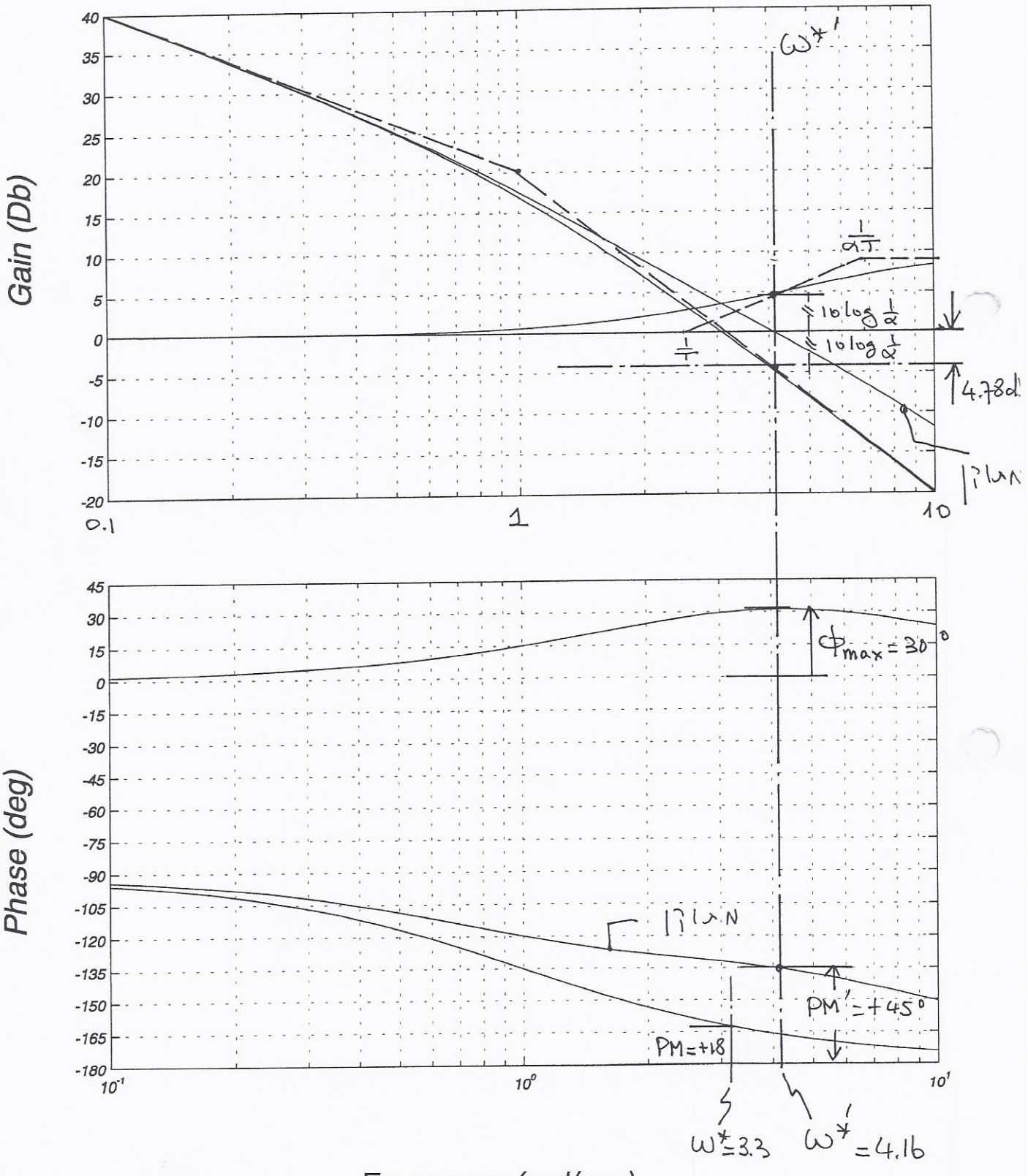
$$\widetilde{G}\widetilde{H} = a\,\frac{(s^2+s+4)}{s[s^2+2s+4]}$$



$G(s)H(s)=10/[s(s+1)]$

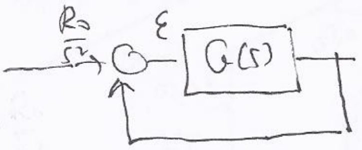

Gain (Db)
40
35
30
25
20
15
10
5
0
-5
-10
-15
-20
0.1
1
10
Phase (deg)
-90
-105
-120
-135
-150
-165
-180
10^{-1}
10^{0}
10^{1}
PM+18°
$\omega^* = 3.16$
Frequency (rad/sec)

With Correction Network (LEAD)



[illegible]

מטרה: לקיים דרישות סותרות [illegible]



: Nyquist

$$\varepsilon_{ss}^{v} = \lim_{s\to 0} s\cdot\frac{R_0}{s^2}\frac{1}{1+G}$$

$$= \lim_{s\to 0}\frac{R_0}{(s+sG)}$$

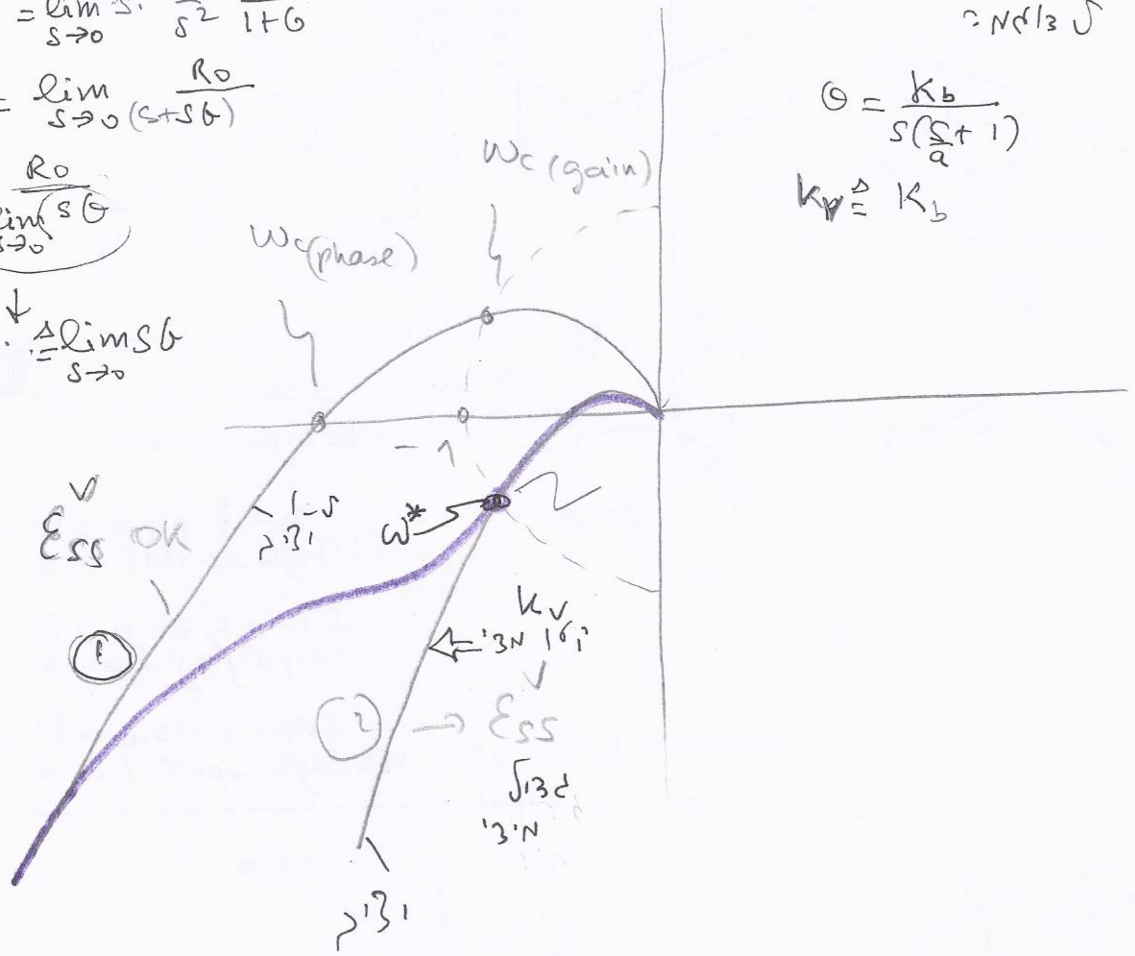
$$= \frac{R_0}{\lim_{s\to 0} sG}$$

$$K_v \triangleq \lim_{s\to 0} sG$$

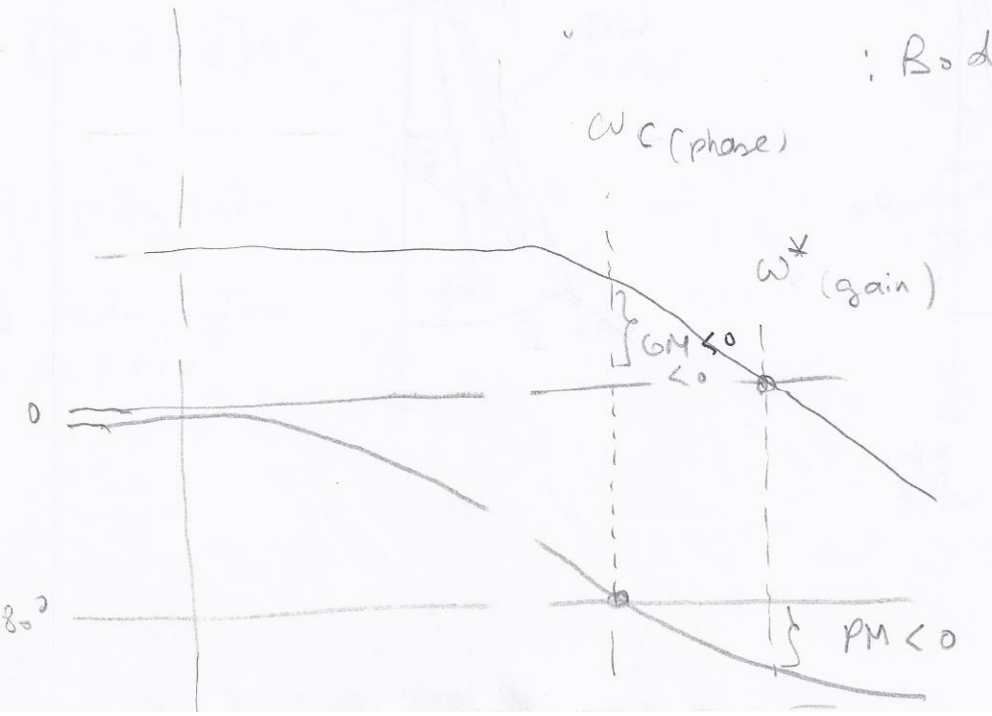
למשל:

$$G = \frac{K_b}{s(\frac{s}{a}+1)}$$

$$K_v \triangleq K_b$$



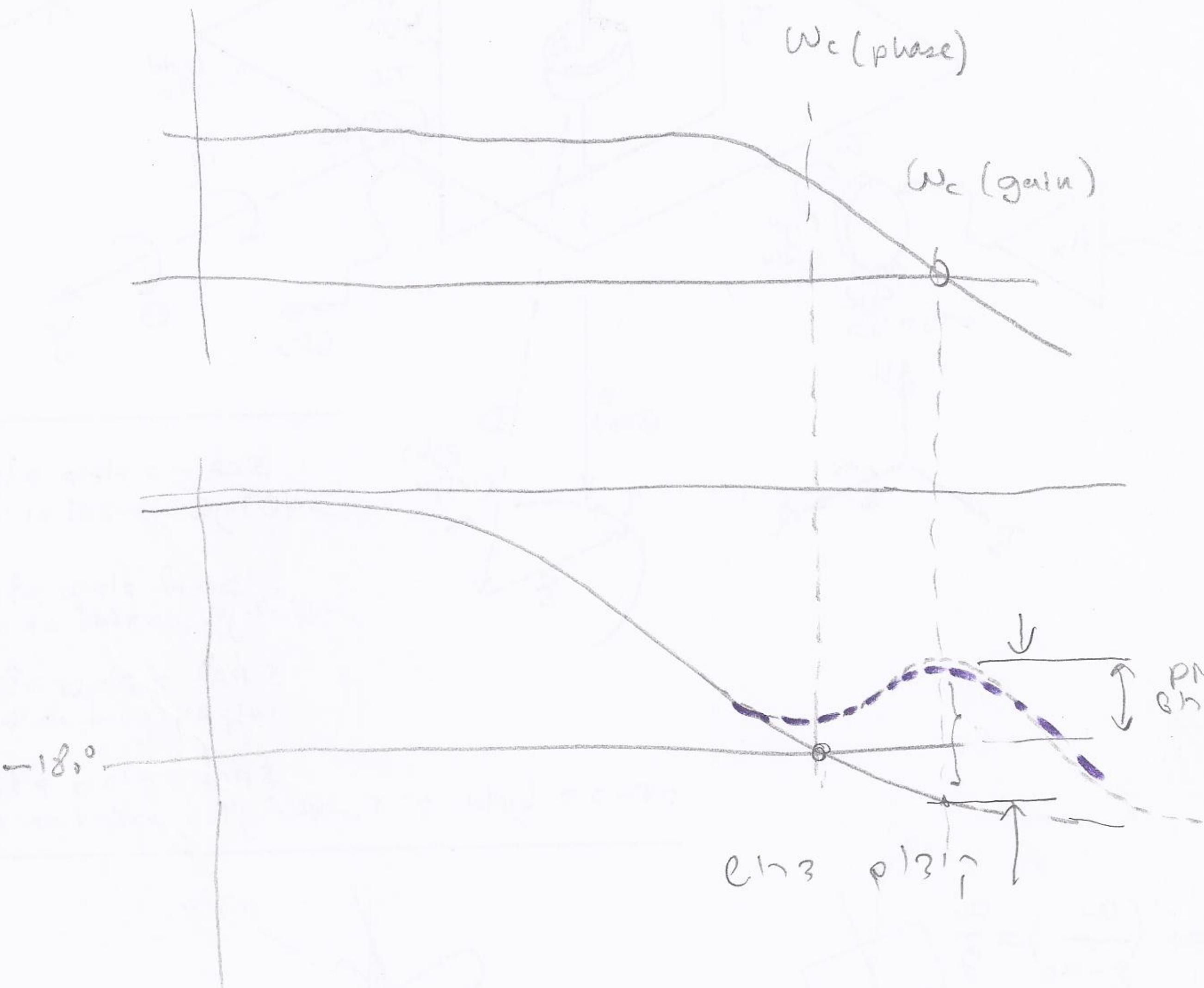
: Bode



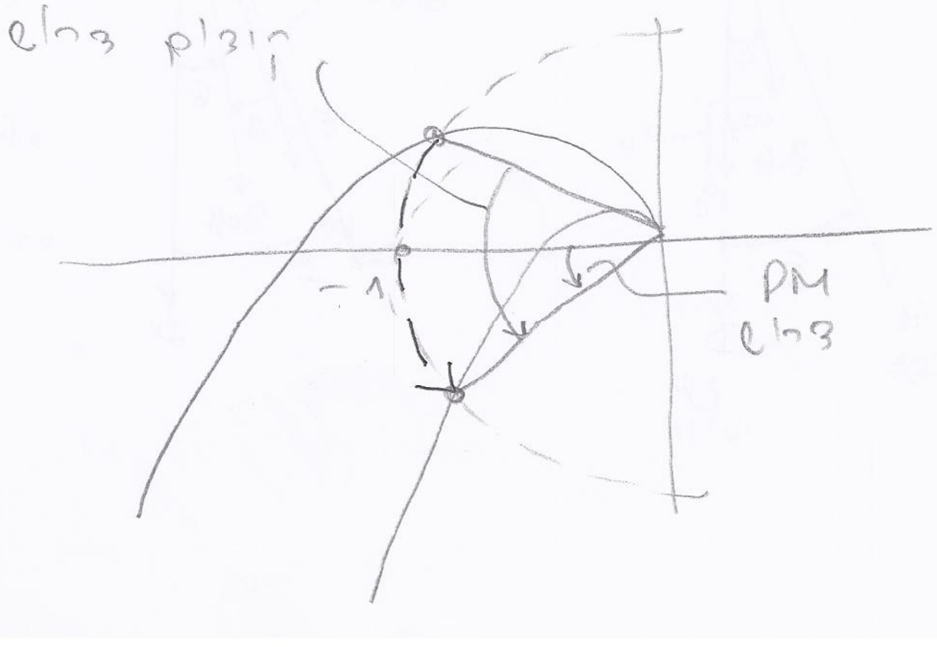
לא יציב

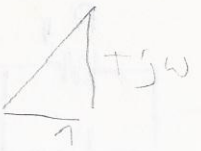
צריכים להגדיל את הפאזה באזור ω

דוגמא : $\omega_c(gain)$:



ב Nyquist :





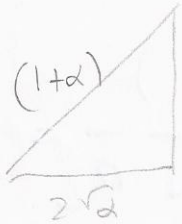
$$\tan^{-1} T\omega - \tan^{-1}(\alpha T\omega)$$

$$\omega = \frac{1}{T\sqrt{\alpha}}$$

$$\phi = \underbrace{\tan^{-1}\left(\frac{1}{\sqrt{\alpha}}\right)}_{\phi_1} - \underbrace{\tan^{-1}\left(\sqrt{\alpha}\right)}_{\phi_2}$$

$$\tan\phi = \tan(\phi_1 - \phi_2) = \frac{\tan\phi_1 - \tan\phi_2}{1 + \tan\phi_1 \tan\phi_2}$$

$$\tan\phi_m = \frac{\frac{1}{\sqrt{\alpha}} - \sqrt{\alpha}}{1 + \frac{1}{\sqrt{\alpha}}\cdot\sqrt{\alpha}} = \frac{1}{2}\cdot\frac{(1-\alpha)}{\sqrt{\alpha}}$$



$$\sqrt{(1-\alpha)^2 + 4\alpha} =$$

$$\sqrt{1 - 2\alpha + \alpha^2 + 4\alpha} =$$

$$\sqrt{\alpha^2 + 2\alpha + 1}$$

$$\sqrt{(\alpha+1)^2} = 1+\alpha$$

$$\boxed{\sin\phi_m = \frac{1-\alpha}{1+\alpha}}$$