$restart:$

Given is the function $h(x) = -\frac{x^3}{x^2-3}$, please analyse the function

Function Analysis

Definition

Let divide $h(x)$ into 2 functions, for the nominator and the denominator $g(x)$

$f := -x^3:$

$g := x^2 - 3:$

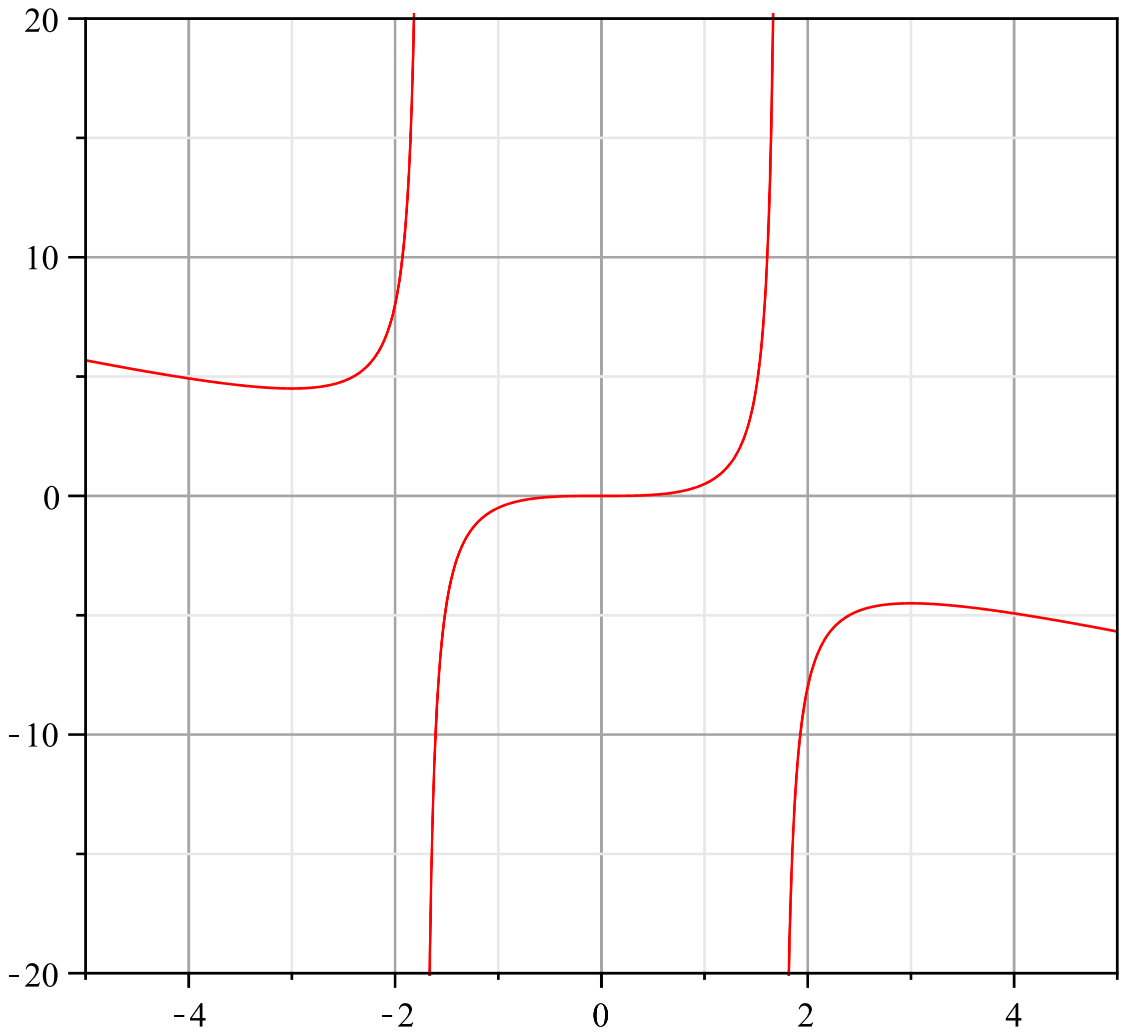
so $h(x)$ is simply the ratio:

$h := \frac{f}{g}$

$$-\frac{x^3}{x^2-3} \qquad \textbf{(1.1.1)}$$

Plot

$plot(h, x = -5..5, y = -20..20, discont = true)$



Poles and Zeros

Zeros are the values of x for which $h(x) = 0$, that is when the nominator $f(x) = 0$

$solve(h = 0, x)$

$$0, 0, 0 \tag{1.3.1}$$

Poles are values of x for which the denominator of $h(x)$, $g(x) = 0$ and hence $h(x)$ is NOT DEFINED!

These values of x are called "singularity point" and a vertical asymptote is suspected to be found there.

$solve(g = 0, x)$

$$\sqrt{3}, -\sqrt{3} \tag{1.3.2}$$

Intersection with the vertical axis

Intersection with the y-axis happens at $x = 0$, hence

$eval(h, x = 0) = 0$

Derivative and Extermum Points

The derivative of $h(x)$ is simply $h' = -\frac{3x^2}{x^2 - 3} + \frac{2x^4}{(x^2 - 3)^2}$

It is better understood after a little simplification
simplify(*expand*(*h*'))

$$-\frac{x^2 (x^2-9)}{(x^2-3)^2} \quad \textbf{(1.5.1)}$$

and further treatment is to factor the nominator
factor(**(1.5.1)**)

$$-\frac{x^2 (x-3)(x+3)}{(x^2-3)^2} \quad \textbf{(1.5.2)}$$

The derivative equals zero at
solve(*h*'= 0, *x*)

$$0, 0, 3, -3 \quad \textbf{(1.5.3)}$$

but be aware, as the last two point are also **singularity points** for the function of the derivative! The examine if an extermum point is a minima or maxima, the sign of the second derivative has to be examined:

$eval(h'', x=-3) = \frac{3}{2}$	positive second order derivative ⇒minima
$eval(h'', x=0) = 0$	Zero on second order derivative ⇒Saddle point
$eval(h'', x=3) = -\frac{3}{2}$	negative second order derivative ⇒maxima

Increasing and Decreasing Ranges

The function $h(x)$ is increasing, as long as the derivative is positive:
solve(*h*'> 0, *x*)

$$RealRange\left(Open(-3), Open\left(-\sqrt{3}\right)\right), RealRange\left(Open\left(-\sqrt{3}\right), Open(0)\right), RealRange\left(Open(0), Open\left(\sqrt{3}\right)\right), RealRange\left(Open\left(\sqrt{3}\right), Open(3)\right) \quad \textbf{(1.6.1)}$$

The function $h(x)$ is decreasing, as long as the derivative is negaitive:
solve(*h*'< 0, *x*)

$$RealRange(-\infty, Open(-3)), RealRange(Open(3), \infty) \quad \textbf{(1.6.2)}$$