Maple notes for introductory control systems

Thomas Lee Ph.D.
Adjunct Professor, Systems Design Engineering, University of Waterloo

Table of Contents

▼ 1. How to use this guide

This guide provides concise examples for all major topics in an introductory course for control systems. In addition, it provides essential Maple concepts techniques that will help in most topics in engineering.

Overall, this guide is intended to help you build your Maple proficiency in a way that helps you enhance your mastery and comprehension of the many complex topics in the course. Maple is a professional tool for performing a wide range of math operations that would normally take you minutes and sometimes hours to do. With the right Maple skills, you will be able to streamline your assignment, and project work. More importantly, it will save you time so that you can redirect your study efforts to understanding the course concepts rather than chasing down a math mistake. In this way, Maple is NOT a substitute for doing your course work. Even if Maple can do a problem instantly, you will still need to build up manual proficiency in the basic techniques of this course as you will need these basic skills to succeed in tests and exams where you will not have access. The best way to use Maple is to buy yourself time to check answers that you try by hand, do a few more problems without spending huge amounts of additional time, and try some textbook problems that do not have answers in the back of the book.

Organization of the guide

- Sections 1.0 and 2.0 provide the basic Maple techniques that are useful for any topic.
- Sections 3.0 to 7.0 provide a series of short examples for all of the major topic areas encountered in a typical course. You will be able to look up the topic of interest and find a quick example to help you use Maple for that topic.

It is strongly recommended that you work through sections 1.0 and 2.0 first. Maple is a sophisticated system with a huge range of options. The first sections provide essential tips and recommendations to help you sort out all of the possibilities.

Working through the examples

This guide is written as a Maple document file, which means all math contained here is live and you can do things directly in this document. The recommended approach however is to work with a blank document file and work through the examples separately. Although the Maple document environment lets you execute all of the example within this document, you can quickly make a real mess of the document content by working directly with the original material. Working through on a separate document is also a good way to practice your equation entry. Once you master Maple's ability to compose and displace complicated math, you will find that Maple's math environment is one of the fastest and most intuitive ways for working with large amounts of mathematical information.

Explanation of symbols

Throughout the guide you will see three symbols attached to various portions of the content.

 This denotes "Clickable" content and refers to Maple techniques to use the mouse, menus, and other more interactive ways of solving problems. In many ways, this is the modern Maple approach.

 This denotes the command approach to Maple problem-solving. Sometimes, it is not possible to compute certain complex things using the clickable methods. For those cases, the examples rely on Maple's extensive range of commands. This is the traditional way of using Maple and one that you might see in a lot of older books and guides.

 This symbol denotes "Exploration" and marks special content designed to help you explore particular complex topics.

The vast majority of this guide is focused on developing your proficiency in using this tool for the topic area. The exploration tools are designed to help you investigate interactively certain key concepts that would be very difficult to show manually.

For more information on the nature of the content in this guide, see Clickable vs. the Command section.

Go back to Table of Contents

1.1 About Maple

Maple is the original system produced by Maplesoft for a broad range of mathematical problem-solving. It is best known for its "symbolic" computation. Maple can compute full mathematical solutions to problems. For example for $\int \frac{x}{1+x^2}\,dx$ it can calculate $\frac{\ln(1+x^2)}{2}$ as the answer. Most math systems would only be able to do this problem through "numerical methods" which produces a simple approximation for the answer that is often less useful and less efficient than the full mathematical answer. Maple was invented at the University of Waterloo in the 1980's and was commercialized by Maplesoft. Today, Maple is used by millions of professionals in industry, university, and research institutions around the world.

MapleSim is the newest system from Maplesoft that applies the symbolic capabilities of Maple specifically for engineering modeling. It offers a nice graphical user interface to specify complete physical systems and then will use advanced symbolic techniques to produce high-performance mathematical models of the system. In control systems, it will help you define system models and investigate control strategies.

Go back to Table of Contents

1.2 About Maple and MATLAB®

In many textbooks, there will be references to the MATLAB math system and most will even provide short examples of important tasks. Maple, in many ways, is a superset of the MATLAB system. It provides all of the basic functions of MATLAB but Maple offers more functions for symbolic calculations (and numeric calculations). This option for symbolic makes Maple much more suitable for fundamental and theoretical concepts of a course like Control Systems. Also, the Maple user interface is much better suited for mathematical work.

Consider the following example for the definition of a transfer function.

MATLAB	**Maple**

`numb = [0 0 100];`
`denb = [1 10.1 101];`
`sysb = tf(numb, denb);`
	$sysb := \frac{100}{s^2 + 10.1\,s + 101}$

MATLAB is principally a programming language and as such will require mathematics in computer-friendly forms. Maple offers several options to represent mathematics in more intuitive ways making it much easier to work with.

In terms of a Control Systems course, this guide was designed to offer a rich set of examples the go well beyond the selection of MATLAB examples that are typically included in standard texts.

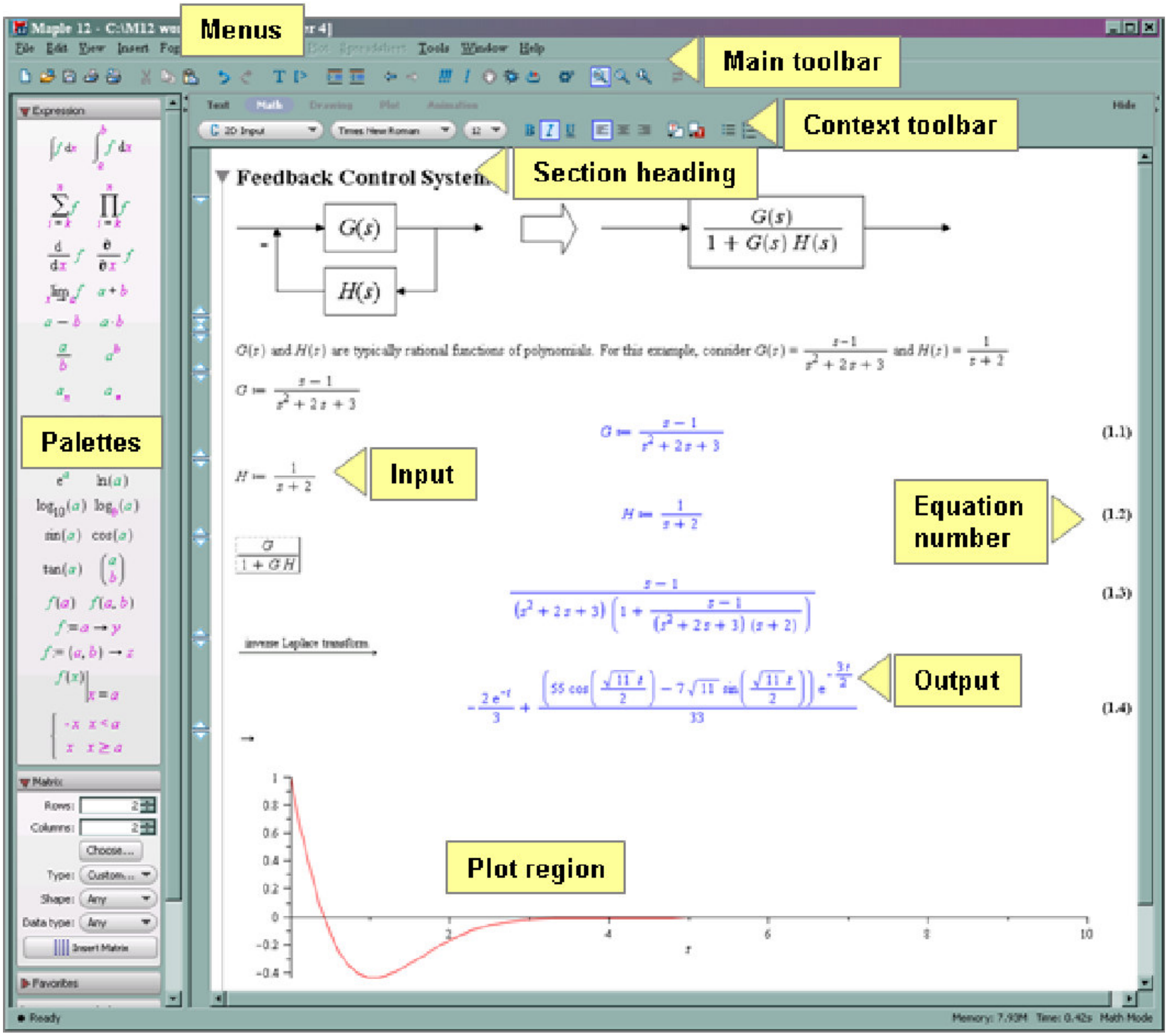
Go back to Table of Contents

2. Maple fundamentals

Maple is an extensive system and offers many ways of performing the same task. The purpose of this guide is to provide simple, easy-to-repeat ways of solving common problems and performing common operations. But it is, in no way, the only approach. After becoming comfortable with the various techniques shown in this guide, you may wish to browse help system to learn about alternate techniques.

2.1 The Maple environment

Maple default user interface is a typical Windows or Mac-like environment.

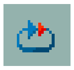
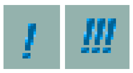
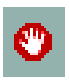
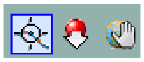


The following are the main parts:

- The main document which is the primary working area. In fact, if you think about it as Microsoft Word with all sorts of extra tools for math and plotting, things make a lot more sense.

- Palettes. Collection of special math tools and symbols that you can drag and drop into your document space. The most important ones will be *Expressions, Matrix, Common Symbols,* and *Greek*. There are actually about 30 available palettes. To see them, right-click on a blank area of the palette region and select *Arrange Palettes*.

• Toolbars. There are two principal toolbars. The standard toolbar, by default, has things like *Save*, *Zoom*, etc., and several important Maple-specific items. The context-sensitive toolbar contains tools and operations that are specific to what you have clicked (e.g. math, a plot, an animation, etc.). The toolbar will change with each context. The following table outlines some of the more important toolbar buttons unique to Maple.

	Restart. Click this if you want to clear the memory and reset all variable definitions. It is a good idea to do this before every example or problem to make sure you are not carrying any definitions from a previous calculation.
	Insert a *Text Region* after the cursor. This puts a blank paragraph and puts Maple into *Text* mode. In this mode you cannot enter math. To go into *Math* mode, press [F5].
	Single exclamation execute all Maple calculations in a selection. The three exclamations is for executing everything in the document.
	Interrupt computation. Sometimes, large problems take longer than you might wish. Pressing this button will cancel the computation.
	Three useful buttons from the *Plot* context toolbar. Respectively, they are the pointer/selection mode (default), scaling (zooming) mode, and panning (shifting) mode.
	Main slider for the *Animation* context toolbar. The slider is used to manually control the animation. Other buttons allow for automatic playing.

• Input and output. Any math that is displayed in real mathematical form is theoretically live and can be executed. However, in real life, you will want some math to be executed while others are used in sentences for explanation purposes. In this guide, for almost all examples, the executable math is in a separate line of its own. This is considered the *input*. The *output* is then displayed in blue and typically centered on the screen.

• Plot and graphics regions. Plots, animations, static images, and other elements can be included into a document.

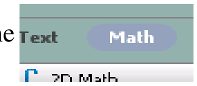
• Sections and subsections. You can group content into collapsible sections and subsections. Click on the little grey triangle to expand or collapse.

[Go back to Table of Contents](#)

2.2 Before you start ...

Remembering a few simple rules even before you have a clear idea of what this system does will help the first learning steps a lot easier. Most beginning Maple users first stumble here and there with the user interface. The following tips will help smooth out the first hours. Simply memorize these tips for now.

• Maple has modes. This means if you are not in the right mode, you will not be able to access certain features. To work with examples in this guide, the most important mode is the *Math* mode -- i.e. the mode where you can enter and work directly with math expressions. There are more details on this topic in *Formats and modes*.

- If your math looks like this: s+a/s^2 , press [F5] or press the 

button. You got yourself into Text mode when you should be in Math mode.

- For most cases, the opposite of Math mode is *Text* mode which is used for entering regular paragraphs of text. In Text mode, Maple will not automatically introduce special formatting to deal with math. If your text looks like this: **if** *your text looks*, press [F5] or press the 

button. You got yourself into Math mode when you should be in Text mode.

- If you are in an exponent or a denominator and you want to get back to the main line of your math, press the right cursor key [→].

- If you cannot seem to delete something through [Backspace] or [Del], try placing the cursor on the desired line and press [Ctl][Del] which is Maple's "Super Delete" for complex information.

Go back to Table of Contents

2.3 Clickable vs. the Command

Maple has two basic approaches to problem solving: the "clickable" approach and the "command" approach.

 The clickable approach is based on the mouse: i.e. using menus, drag-and-drop, clicking on icons, etc. It is highly interactive and is very convenient for quick problem-solving and interactive manipulation and exploration of mathematical topics.

 The command approach is based on a large set of precise commands or function names that you type in to perform specific tasks. This is the preferred way for advanced work and programming.

This guide assumes that you may want to use both methods depending on circumstances and provides, in most cases, both clickable and command notation options. The mouse and keyboard symbols shown here will indicate whether the style of the example is clickable or command.

Go back to Table of Contents

2.4 Doing math with Maple

2.4.1 Entering Maple expressions

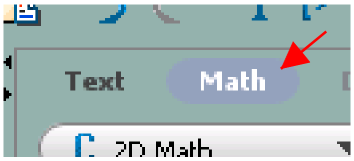
In Maple, mathematical expressions, such as $\frac{s+a}{s^2+s+\sqrt{2}}$, can be entered in a natural 2-D notation. These expressions can be used for either clickable or command techniques. Alternatively, if you are progressing to advanced programming, you may use text style 1D notation (`(s+a)/(s^2+s+sqrt(2))`). The command examples in this guide generally use the 2D notation.

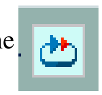
Go back to Table of Contents

2.4.1.1 Example: $\dfrac{s+a}{s^2+s+\sqrt{2}}$

Ensure that you are in Math mode, and follow the steps below to enter the expression $\dfrac{s+a}{s^2+2\,s+\sqrt{2}}$.

To begin entering a 2-D mathematical expression, ensure that you are in *Math mode*. In this mode, the cursor is slanted. If the cursor is vertical, you are in *Text mode* and you cannot do calculations or compose complex math expressions. To switch between Math and Text mode, press the appropriate button on the top left of your workspace or press [F5] to toggle between the two modes.



Before you start, you should reset Maple's memory by clicking the  button on your Maple toolbar.

 Step	Action	Result
1	Enter the numerator $s+a$	$s+a$
2	Highlight $s+a$ and type `[/]` (slash). The slash denotes division.	$\dfrac{s+a}{}$
3	Begin entering denominator. Type s	$\dfrac{s+a}{s}$
4	To enter an exponent, type the common power character `[^]`. The cursor is now in the exponent position. Type 2.	$\dfrac{s+a}{s^2}$
5	To bring the cursor back onto the base line, press the right arrow [→] key on your keyboard.	$\dfrac{s+a}{s^2}$
6	Type $+s+$. There is no need to type a space, as Maple automatically adjusts spacing for you.	$\dfrac{s+a}{s^2+s+}$
7	To enter a square root, open the expression palette on left, and find the [$\sqrt{a}$] symbol. Click on this. This will insert the symbol into the expression. The green a will be highlighted.	$\dfrac{s+a}{s^2+s+\sqrt{a}}$
8	With the a highlighted, type 2. Then press the right arrow [→] key to return to the baseline	$\dfrac{s+a}{s^2+s+\sqrt{2}}$
9	Press return to enter the expression into memory. Maple will return an "equation number" on the right (note, your equation number will likely be different than in this example). The number is useful later to refer to this expression in a calculation.	$\dfrac{s+a}{s^2+s+\sqrt{2}}$ $\dfrac{s+a}{s^2+s+\sqrt{2}}$ **(2.4.1.1.1)**

Alternative to Step 1: Find the [$\frac{a}{b}$] symbol on the expression palette and replace the numerator and denominator with the appropriate expressions as above.

Alternative to Step 7: For the square root, type `[s][q][r][t]` and then `[ctl][space]` (keep `[ctl]` pressed). You will get a menu of possible Maple symbols beginning with the letters sqrt. Choose the $\sqrt{x}$ option.

Go back to Table of Contents

2.4.1.2 Example: $s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2$

Ensure that you are in Math mode, and follow the steps below to enter the expression

 **Step**	**Action**	**Result**
1	Enter the numerator $s + 2$. Type an additional [space]	$s + 2$
2	For the Greek letter ξ (xi), type [x][i] then [ctl][space]. The letters x and i will immediately switch to the Greek letter. In this case, there is only one option in Maple for the letters x and i so the change is automatic. Type an additional [space]	$s + 2\,\xi$
3	For the product $\omega_0^2\,s$: • Type [o][m][e] then [ctl][space]. Switches immediately to the Greek ω. • Type [_] (underscore) which puts you in subscript mode. Type [0] (zero) • press the right arrow [→] key to return to the baseline • Type [^] for the exponential then [→] to return to the baseline. • Enter s to complete the product.	$s + 2\,\xi\,\omega_0^2\,s$
4	Complete the expression using the same technique for the ω_0^2 term.	$s + 2\,\xi\,\omega_0^2\,s + \omega_0^2$
5	Press return to enter into memory.	$s + 2\,\xi\,\omega_0^2\,s + \omega_0^2$ $s + 2\,\xi\,\omega_0^2\,s + \omega_0^2$ **(2.4.1.2.1)**

Alternative for Greek letters: All Greek letters are also available through the Greek palette. If you do not see a Greek palette, right click in the palette area, choose "Show Palette" in the menu then choose Greek.

Alternative for multiplication: In this example, all multiplication was denoted by [space]. You can also explicitly specify multiplication by the [*] character. On the display, it will show as a dot (e.g. $2\cdot\xi$).

Go back to Table of Contents

2.4.2 Doing some math on expressions

2.4.2.1 Differentiation using menus and palettes

The easiest way of performing basic math operations is via the right-click (also known as "context-sensitive") menus. Simply right-click (if on a Windows computer) on an expression and Maple will automatically offer a range of mathematical operations suitable for that expression. Given an expression $s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2$, calculate the derivative $\frac{d}{d\,\xi}$ and $\frac{d}{d\,\omega_0}$

Step	Action	Result
1	Enter expression $s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2$. Press [Enter]. The blue expression that Maple returns is referred to as the "output".	$s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2$ $s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2$ **(2.4.2.1.1)**
2	Place the mouse somewhere on the output and press the right mouse button. You will see a menu. Choose *Differentiate*, then choose *xi*. Maple returns the answer and adds a little note to indicate what just happened. Check the answer with your intuition.	$s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2$ $s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2$ **(2.4.2.1.2)** differentiate w.r.t. xi → $2\,\omega_0^2\,s$ **(2.4.2.1.3)**
3	Make a copy of the expression to a new line. Enter it. Right-click and differentiate with respect to omega[0]. This is how Maple represents ω_0 within text menus.	$s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2$ $s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2$ **(2.4.2.1.4)** differentiate w.r.t. omega[0] → $4\,\xi\,\omega_0\,s + 2\,\omega_0$ **(2.4.2.1.5)**

Alternate method for differentiation: In the Expression palette, there is a symbol $\frac{d}{dx}f$. Click on this symbol and you'll see a template for a single differentiation. You can now replace the *x* with ξ and *f* with $\left(s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2\right)$. Note that the brackets around the expression was intentional to avoid possible misinterpretation: i.e. $\frac{d}{d\,\xi}\left(s^2 + 2\,\xi\,\omega_0^2\,s + \omega_0^2\right)$ vs. $\left(\frac{d}{d\,\xi}s^2\right) + 2\,\xi\,\omega_0^2\,s + \omega_0^2$.

Go back to Table of Contents

2.4.2.2 Differentiation using commands

Consider the same example of $\frac{d}{d\,\xi}$ and $\frac{d}{d\,\omega_0}$. If you're working with commands, the following are the equivalent commands,

$diff\left(s^2+2\,\xi\,\omega_0^2\,s+\omega_0^2,\xi\right)$

$$2\,\omega_0^2\,s \tag{2.4.2.2.1}$$

$diff\left(s^2+2\,\xi\,\omega_0^2\,s+\omega_0^2,\omega_0\right)$

$$4\,\xi\,\omega_0\,s+2\,\omega_0 \tag{2.4.2.2.2}$$

Go back to Table of Contents

2.4.2.3 Other math operations on common expressions

Operation	**Clickable** 
Given $\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$, simplify the expression.	Use right click menu and choose *Simplify* → *Simplify*. $\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$ $\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$ (2.4.2.3.1) $\overset{\text{simplify}}{=}$ $\frac{3}{(s+2)(3\,s+1)}$ (2.4.2.3.2)
Using the result from the above, • extract denominator • expand denominator • factor the expanded denominator	Use the right click menu for all operations. All operations are at the top level of menu. $\frac{3}{(s+2)(3\,s+1)}$ $\frac{3}{(s+2)(3\,s+1)}$ (2.4.2.3.3) $\xrightarrow{\text{denominator}}$ $(s+2)(3\,s+1)$ (2.4.2.3.4) $\overset{\text{expand}}{=}$ $3\,s^2+7\,s+2$ (2.4.2.3.5) $\overset{\text{factor}}{=}$ $(s+2)(3\,s+1)$ (2.4.2.3.6)

Command version

$simplify\left(\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}\right)$

$$\frac{3}{(s+2)(3\,s+1)} \tag{2.4.2.3.1.1}$$

$denom\left(\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}\right)$

$$(s+2)(3\,s+1) \tag{2.4.2.3.1.2}$$

$expand(\,(s+2)\,(3\,s+1)\,)$

$$3\,s^2+7\,s+2 \qquad \textbf{(2.4.2.3.1.3)}$$

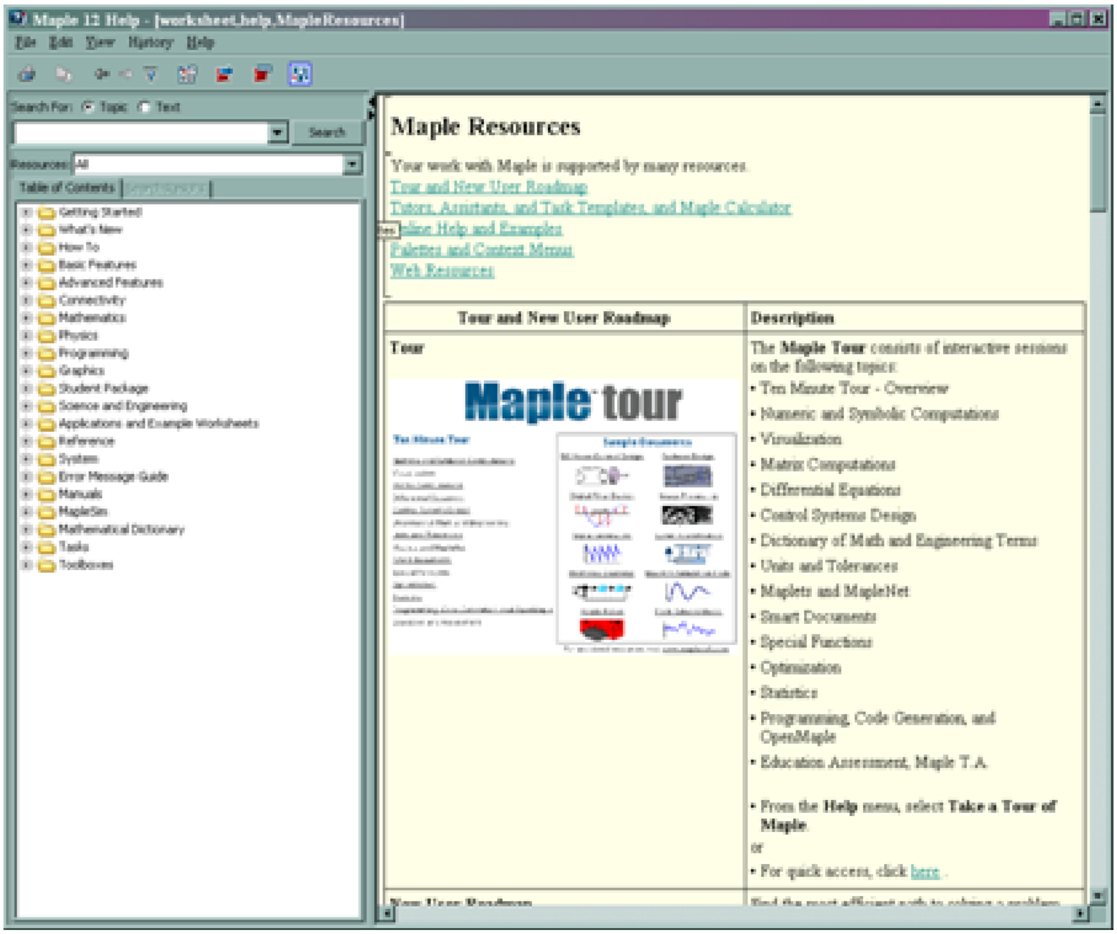
$factor(3\,s^2+7\,s+2)$

$$(s+2)\,(3\,s+1) \qquad \textbf{(2.4.2.3.1.4)}$$

[Go back to Table of Contents](#)

2.4.3 The Maple Help system

Maple has an extensive help system. To browse the system, simply go to the *Help* menu and select one of the various options. If you are used to using commands, you can also use the question mark. For example ?*dsolve* will bring up the specific help page for the differential equation solving command *dsolve*. This is useful if you want more detailed information on a command that you see in this guide.



Each Help page contains in-depth technical information on the topic as well as examples that you can copy and paste and try out. In some cases the Help system will lead you to a full Maple document with lots of examples for you to try out.

The Help system also includes a full dictionary of math terms. You can access the dictionary either through *Mathematical Dictionary* in the folder pane of the Help browser. If you use the search dictionary on a term like *zeta*, you will get a variety of search hits as both Maple and the math dictionary has entries for the item. The icons beside the hits will indicate what kind of entries they are. Often you will also see cross-referencing from a dictionary page to relevant Maple topics and commands.
[Go back to Table of Contents](#)

2.4.4 Evaluating expressions for different values

Given a Maple expression, you can substitute variable or parameter values. There are many ways to do this and with experience, you will develop your own preferences.

2.4.4.1 Right-click menu

Evaluate the expression $e^{-a\,t}\cos(b\,t) + c$, for different values of the unknowns.

Operation	**Clickable** 
To form $e^{-a\,t}\cos(b\,t) + c$, the trickiest part will be to form the exponential. **You cannot simply type "e" and raise it to a power**. Maple will interpret "e" as a variable name "e" and not the special number (Euler number 2.718...). Type [e][x][p][ctl][space]. Choose e^x. Type in the rest taking care of the multiplication sign between *b* and *t* and making sure there is no space between cos and the bracket. Press [enter].	$e^{-a\,t}\cos(b\cdot t) + c$ $e^{-a\,t}\cos(b\,t) + c$ **(2.4.4.1.1)**
Right-click on the output and choose *Evaluate at a point*. You will get a dialog box where you can specify values for the unknowns. The result on the right is for $a = \frac{1}{2}$ and $c = 2$.	$e^{-a\,t}\cos(b\cdot t) + c$ $e^{-a\,t}\cos(b\,t) + c$ **(2.4.4.1.2)** $\xrightarrow{\text{evaluate at point}}$ $e^{-\frac{t}{2}}\cos(b\,t) + 2$ **(2.4.4.1.3)**

Command version 

$$eval\left(e^{-a\,t}\cos(b\cdot t) + c, \left[a = \frac{1}{2}, c = 2\right]\right)$$

$$e^{-\frac{t}{2}}\cos(b\,t) + 2 \qquad \textbf{(2.4.4.1.1.1)}$$

Go back to Table of Contents

2.4.4.2 Palette

Evaluate the expression $e^{-a\,t}\cos(b\,t) + c$ again for $a = \frac{1}{2}$ and $c = 2$. This time use the palette. For this, it is useful to consider the mathematical form of this task: $\left. e^{-a\,t}\cos(b\,t) + c \right|_{a = \frac{1}{2}, c = 2}$

Operation	Clickable 
From the Expression palette, choose, $f(x)\Big|_{x=a}$ (near the bottom). Replace $f(x)$ with $e^{-a\,t}\cos(b\cdot t)+c$ and $x=a$ with $a=\frac{1}{2}, c=2$.	$e^{-a\,t}\cos(b\cdot t)+c\Big|_{a=\frac{1}{2}, c=2}$ $e^{-\frac{t}{2}}\cos(b\,t)+2$ **(2.4.4.2.1)**
One trick that makes it easy to compose the above in cases when you already have expressions defined in your document and you want to do complex assignments is to first replace the $f(x)$ and $x=a$ with empty brackets (with 3 spaces in between) and then copy and paste your existing expressions.	$(\quad)\Big|_{(\quad)}$

Command version 

$$eval\left(e^{-a\,t}\cos(b\cdot t)+c, \left[a=\frac{1}{2}, c=2\right]\right)$$

$$e^{-\frac{t}{2}}\cos(b\,t)+2 \quad \textbf{(2.4.4.2.1.1)}$$

[Go back to Table of Contents]

2.4.4.3 Equation numbers

Maple automatically assigns an equation number when you press return to enter an expression or compute an answer. The number in the round brackets is the equations number. The number will have more dots if your operation is inside nested sections (denoted by the little inverted grey triangles). For example, evaluate the expression $\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$ in two steps for $s=k\cdot\omega$.

Operation	Clickable 
Enter $k\cdot\omega$.	$k\cdot\omega$ $k\,\omega$ **(2.4.4.3.1)**
Enter $\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$ but at every instance of s, press [ctl][l] (lowercase L), enter the equation number from first step (i.e. in this example, **(2.4.4.3.1)**).	$\frac{1}{(\textbf{(2.4.4.3.1)}+2)\left(\textbf{(2.4.4.3.1)}+\frac{1}{3}\right)}$ $\frac{1}{(k\,\omega+2)\left(k\,\omega+\frac{1}{3}\right)}$ **(2.4.4.3.2)**

Command version

$$eval\left(\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}, [s = k\,\omega]\right)$$

$$\frac{1}{(k\,\omega+2)\left(k\,\omega+\frac{1}{3}\right)} \qquad (2.4.4.3.1.1)$$

Go back to Table of Contents

2.4.4.4 Assigning variable names

This is the traditional way of working with expressions and values. By assigning variable names, you can easily manage large sets of equations and variables. For example, the statement

$f := \sin(x)$

$$f := \sin(x) \qquad (2.4.4.4.1)$$

reads, "let the variable f have the expression $\sin(x)$". The special Maple symbol $:=$ ([:][=], no space) is called the assignment operator. The assignment operator is very common when you work with commands. However when you are using the clickable features, you can still perform assignments through the menu. The following shows the menu-based assignment.

<table>
<tr><th>Operation</th><th>Clickable</th><th>Command version</th></tr>
<tr><td>Define an expression and give it the name expr. In clickable mode, you right click and select Assign to a name. Note that Maple returns the same expression and a little note indicating that a name has been assigned.</td><td>$\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$
$\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$ (2.4.4.4.2)
$\xrightarrow{\text{assign to a name}}$
$\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$ (2.4.4.4.3)</td><td>$expr := \frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$
$\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$ (2.4.4.4.4)</td></tr>
<tr><td>Simplify expr and name the result exprsimp. In clickable mode, right click and choose Simplify → Simplify.</td><td>simplify
$\frac{3}{(s+2)\,(3\,s+1)}$ (2.4.4.4.5)
$\xrightarrow{\text{assign to a name}}$
$\frac{3}{(s+2)\,(3\,s+1)}$ (2.4.4.4.6)</td><td>$exprsimp := simplify(expr)$
$\frac{3}{(s+2)\,(3\,s+1)}$ (2.4.4.4.7)</td></tr>
</table>

enominator of *exprsimp* and give it the name *den*. In clickable mode, right click and choose *Denominator.*	$\xrightarrow{\text{denominator}}$ $(s+2)\ (3\,s+1)$ **(2.4.4.4.8)** $\xrightarrow{\text{assign to a name}}$ $(s+2)\ (3\,s+1)$ **(2.4.4.4.9)**	$den := denom(exprsimp)$ $(s+2)\ (3\,s+1)$ **(2.4.4.4.10)**
To see the value or the assigned expression, simply type the name of the variable and press [Enter].	$expr$ $\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$ **(2.4.4.4.11)**	$expr$ $\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}$ **(2.4.4.4.12)**
You can also string together several variables and have it displayed at once. Youcan also use variable in other formulas and expressions.	$expr, exprsimp, den$ $\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}, \frac{3}{(s+2)\,(3\,s+1)}, (s+2)\,(3\,s+1)$ **(2.4.4.4.13)** $\frac{3}{den}$ $\frac{3}{(s+2)\ (3\,s+1)}$ **(2.4.4.4.14)**	$expr, exprsimp, den$ $\frac{1}{(s+2)\left(s+\frac{1}{3}\right)}, \frac{3}{(s+2)\,(3\,s+1)}, (s+2)\,(3\,s+1)$ **(2.4.4.4.15)** $\frac{3}{den}$ $\frac{3}{(s+2)\ (3\,s+1)}$ **(2.4.4.4.16)**

Once you get to the point of using a larger number of variable names, the advantage of commands over purely clickable approaches become obvious: fewer steps and easier to organize complex problems.

The clickable approach is best for short calculations with relatively fewer steps or where you want to make your math look as close as possible to formal math as found in books and papers. For multi-step problems, a good grasp of Maple's command techniques is extremely helpful.

Go back to Table of Contents

2.4.5 Defining functions $f(x)$

When you have mathematical expression, it is common to want to evaluate the function for many different points. For example, we want to define a function $f = e^{-t}\cos(2\cdot t)$ and you want to calculate $\left(f(1), f(2), f(2\,\pi),\right.$ etc.). Although, it is easy to evaluate Maple expressions for specific values (we have seen several techniques), if you have many such substitutions, you would probably want to have a proper "function" for the calculation.

Consider the following sequence of Maple operations,

$f := e^{-t}\cos(2\cdot t)$

$$e^{-t}\cos(2\,t) \qquad \textbf{(2.4.5.1)}$$

$f\Big|_{t=1}$

$$e^{-1}\cos(2) \qquad \textbf{(2.4.5.2)}$$

$eval(f, t=1)$

$$e^{-1}\cos(2) \qquad \textbf{(2.4.5.3)}$$

In both examples, the substitution works as expected. Now try,

$f(1)$

$e^{-t}(1)\cos(2t)(1)$ **(2.4.5.4)**

The result is not familiar to most of us. This is because Maple has advanced programming functionality that prevents simple variables names from being able to flip to a function.

However, once you get past this one aspect, it is quite easy to work with functions quickly and intuitively in Maple. The following are two of the most important ways.

Go back to Table of Contents

2.4.5.1 Operator or mapping notation

This step is useful if you have not specified your expressions yet -- i.e. you are at the beginning of your work.

In mathematics, we often use the "mapping" notation for functions. For example, $(t) \rightarrow e^{-t}\cos(2t)$ reads, t maps to the function $e^{-t}\cos(2t)$. The equivalent can be specified in Maple.

Operation	**Clickable** 
Define a mapping (function) and assign the name *f*. From the expression palette: $f := a \rightarrow y$. Change a and y appropriately. Note: to get π, type [p][i][ctl][space] and select π.	$f := t \rightarrow e^{-t}\cos(2t)$ $t \rightarrow e^{-t}\cos(2t)$ **(2.4.5.1.1)** $f(1)$ $e^{-1}\cos(2)$ **(2.4.5.1.2)** $f(2\pi)$ $e^{-2\pi}$ **(2.4.5.1.3)** $f(1)+f(2\pi)$ $e^{-1}\cos(2)+e^{-2\pi}$ **(2.4.5.1.4)**

Note that there is also a palette option to make a function of 2 variables $f := (a, b) \rightarrow z$.

Command version 

To get the arrow symbol, type [-][>].

$f := (t) \rightarrow e^{-t}\cos(2t)$

$t \rightarrow e^{-t}\cos(2t)$ **(2.4.5.1.1.1)**

$f(1)$

$e^{-1}\cos(2)$ **(2.4.5.1.1.2)**

$f(1)+f(2\pi)$

$e^{-1}\cos(2)+e^{-2\pi}$ **(2.4.5.1.1.3)**

Go back to Table of Contents

2.4.5.2 Converting regular variables and expressions to functions

Sometimes you will have existing expressions assigned to variable names that you want to convert to behave as functions. For simple single variable functions, there is a convenient menu option. For example, right clicking on an expression gives a menu with an option *Conversions* → *Operator* → (variable list).

First, let's look at some clickable approaches to conversion. 

Operation	**Clickable** 
First, define the expression that we wish to make a function. Right click and choose *Convert* → *Operator*. Assign a name *f2* using the right click menu. Once this is done you can now use the new name *f2* as a reusable function.	$e^{-t}\cos(2t)+c$ $e^{-t}\cos(2t)+c$ **(2.4.5.2.1)** $\xrightarrow{\text{convert to operator}}$ $t \to e^{-t}\cos(2t)+c$ **(2.4.5.2.2)** $\xrightarrow{\text{assign to a name}}$ $t \to e^{-t}\cos(2t)+c$ **(2.4.5.2.3)** $f2(2)$ $e^{-2}\cos(4)+c$ **(2.4.5.2.4)**
Similarly, you can use the same technique to define a function of the variable *c*.	$e^{-t}\cos(2t)+c$ $e^{-t}\cos(2t)+c$ **(2.4.5.2.5)** $\xrightarrow{\text{convert to operator}}$ $c \mapsto e^{-t}\cos(2t)+c$ **(2.4.5.2.6)** $\xrightarrow{\text{assign to a name}}$ $c \to e^{-t}\cos(2t)+c$ **(2.4.5.2.7)** $f3(2)$ $e^{-t}\cos(2t)+2$ **(2.4.5.2.8)**

Command version 

Unfortunately, with the menu option, you can only specify functions of one variable. Furthermore, you have the double step of conversion followed by name assignment. A more flexible and convenient approach uses the Maple low-level command called *unapply*( ).

$myexpr := e^{-t}\cos(2t)+c$

$$e^{-t}\cos(2t)+c \qquad \textbf{(2.4.5.2.1.1)}$$

$newfunc := unapply(myexpr, t, c)$

$$(t, c) \to e^{-t}\cos(2t)+c \qquad \textbf{(2.4.5.2.1.2)}$$

$newfunc(1, 3)$

$$e^{-1}\cos(2)+3 \qquad \textbf{(2.4.5.2.1.3)}$$

Go back to Table of Contents

2.4.6 Numerical formats

In the previous example, Maple produced the result of an evaluation as $e^{-1}\cos(2)$. Why does Maple represent answers in a seemingly incomplete way? For example, any calculator could compute $e^{-1}\cos(2)$ as -0.153092.

Maple can also do this easily but by default it does not for two very important reasons:

- $e^{-1}\cos(2)$ is mathematically exact whereas -0.153092 is a 6 decimal place approximation to the exact number. With an exact representation, you avoid round-off error (not keeping enough decimal places to ensure accuracy) which can be a major problem for large, complex problems.
- $e^{-1}\cos(2)$ preserves some information about the origins of the number and how the value is calculated.

In the above case, both *e* and $\cos(2)$ are both irrational numbers (i.e. cannot be represented by finite or repeating decimal numbers). Other numbers that you could be expressed in an exact Maple answer include:

- fractions (e.g. rational finite decimal places like $\frac{1}{2}$ and rational repeating decimals like $\frac{1}{3}$)
- special numbers like π, and *e*
- numbers calculated from special functions (e.g. cos(2), Bessel functions, Gamma functions etc.)
- irrational numbers like roots (e.g. $\sqrt{2}$)

In context of Maple, the opposite of exact numbers is normally referred to as "floating point approximations".

Go back to Table of Contents

2.4.6.1 Converting from exact to decimal representations (floating point)

Consider the expression $\dfrac{3\,e^{-\frac{t}{6}}\sin\left(\frac{\sqrt{35}\,t}{6}\right)\sqrt{35}}{3}$ which is one of the terms to the solution to a linear differential equation calculated by Maple ($\frac{d^2}{dt^2}y(t) + \frac{1}{3}\frac{d}{dt}y(t) + y(t) = \sin(t)$ with 0 initial conditions). The following shows how to convert this expression to floating point.

Operation	**Clickable** 
Copy or enter the solution expression	$\dfrac{3\,e^{-\frac{t}{6}}\sin\left(\frac{\sqrt{35}\,t}{6}\right)\sqrt{35}}{35}$ $\dfrac{3\,e^{-\frac{t}{6}}\sin\left(\frac{\sqrt{35}\,t}{6}\right)\sqrt{35}}{35}$ **(2.4.6.1.1)**
Approximate the exact answer to 5 decimal places	Right click. Choose Approximate → 5 $\dfrac{3\,e^{-\frac{t}{6}}\sin\left(\frac{\sqrt{35}\,t}{6}\right)\sqrt{35}}{35}$ $\dfrac{3\,e^{-\frac{t}{6}}\sin\left(\frac{\sqrt{35}\,t}{6}\right)\sqrt{35}}{35}$ **(2.4.6.1.2)** $\xrightarrow{\text{at 5 digits}}$ $0.50709\,e^{-0.16667\,t}\sin(0.98604\,t)$ **(2.4.6.1.3)**
Calculate for $t = \frac{1}{3}$	$\xrightarrow{\text{at 5 digits}}$ $0.50709\,e^{-0.16667\,t}\sin(0.98604\,t)$ **(2.4.6.1.4)** $\xrightarrow{\text{evaluate at point}}$ 0.1548397660 **(2.4.6.1.5)**

Command version 

$$expr := \frac{3\,e^{-\frac{t}{6}}\sin\left(\frac{\sqrt{35}\,t}{6}\right)\sqrt{35}}{35}$$

$$\frac{3}{35}\,e^{-\frac{1}{6}\,t}\sin\left(\frac{1}{6}\sqrt{35}\,t\right)\sqrt{35} \qquad \textbf{(2.4.6.1.1.1)}$$

$exprfloat := evalf(expr, 5)$

$$0.50709\,e^{-0.16667\,t}\sin(0.98604\,t) \qquad \textbf{(2.4.6.1.1.2)}$$

$eval\left(exprfloat, t = \frac{1}{3}\right)$

$$0.1548397660 \qquad \textbf{(2.4.6.1.1.3)}$$

Go back to Table of Contents

2.4.6.2 Enforcing floating point calculations

If you prefer to always see the floating point versions of expressions, you can express numerical values as decimal numbers (e.g. from 6 to 6.0. Omitting the 0 and using 6. also works) from the beginning and Maple will carry the floating point calculation from that point on.

Operation	**Clickable** 
For the expression, replace numbers with decimal equivalents	$\frac{3.\,e^{-\frac{t}{6.}}\sin\left(\frac{\sqrt{35.}\,t}{6.}\right)\sqrt{35.}}{35.}$ $0.5070925528\,e^{-0.1666666667\,t}\sin(0.9860132974\,t)$ **(2.4.6.2.1)**

Command version 

$$expr := \frac{3.\,e^{-\frac{t}{6.}}\sin\left(\frac{\sqrt{35.}\,t}{6.}\right)\sqrt{35.}}{35.}$$

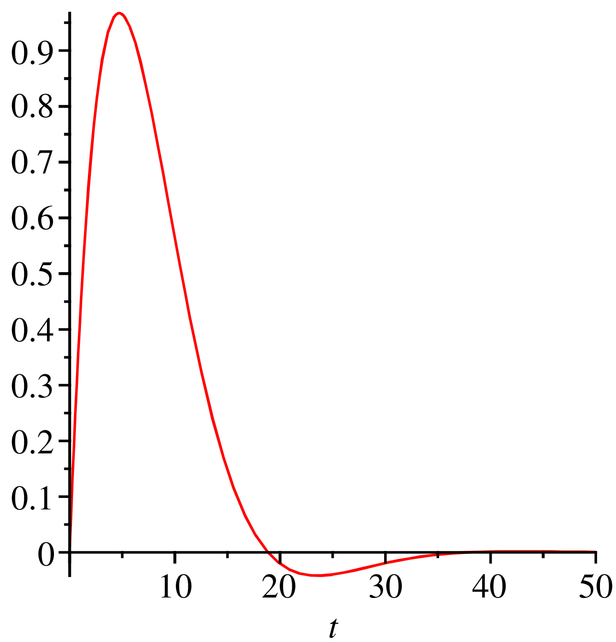
$$expr := 0.5070925528\,e^{-0.1666666667\,t}\sin(0.9860132974\,t) \qquad \textbf{(2.4.6.2.1.1)}$$

Go back to Table of Contents

2.4.7 Plotting

2.4.7.1 Basic 2D plots of math functions $f(t)$

Plot $f(t) = 3\,\mathrm{e}^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)$. One of the easiest ways is to use the Plot Builder from the right-click menu. The Plot Builder offers handy dialog boxes to guide you through the many plotting options.

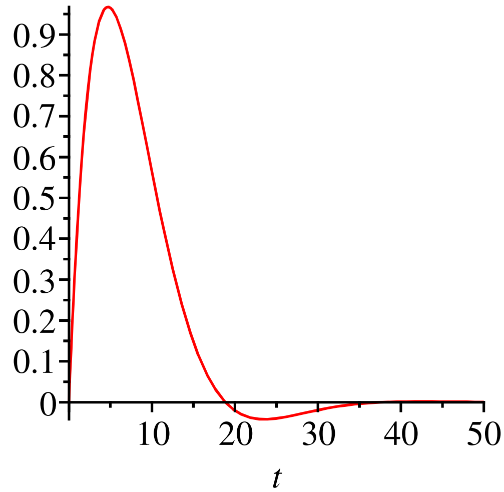
Operation	**Clickable** 
Right-click on expression, choose *Plots* → *Plot Builder*. In the dialog box, change the plot range to 0 to 50 (default is -10 to 10).	$3\,\mathrm{e}^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)$ $3\,\mathrm{e}^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)$ **(2.4.7.1.1)** → 

Command version 

$expr := 3\,\mathrm{e}^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)$

$$3\,\mathrm{e}^{-\frac{1}{6}t}\sin\left(\frac{1}{6}t\right) \quad \textbf{(2.4.7.1.1.1)}$$

$plot(expr, t = 0..50)$

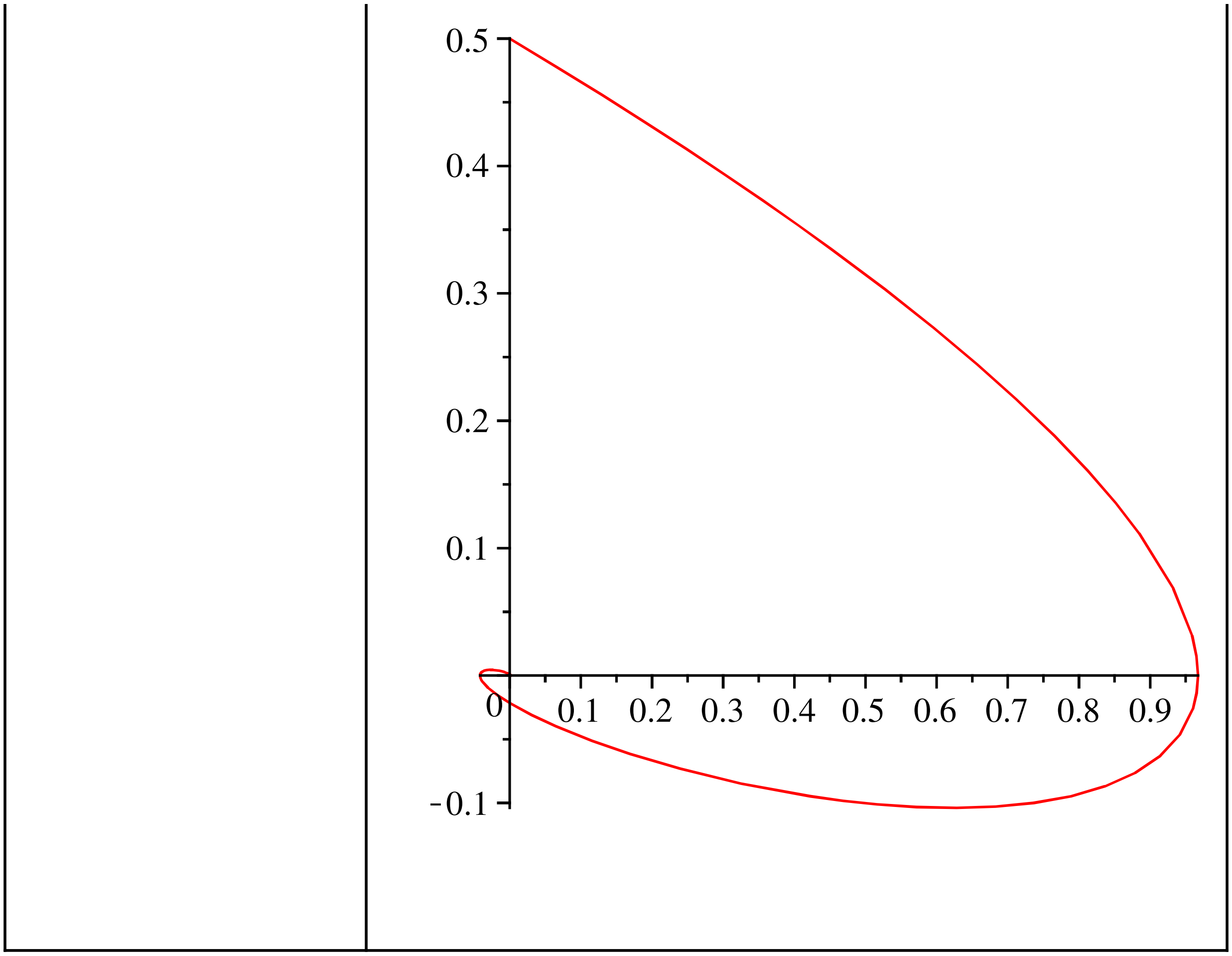


Go back to Table of Contents

2.4.7.2 Parametric (phase plane) plots $(x(t), y(t))$

Plotting parametrically defined curve $(x(t), y(t))$ (i.e. the coordinates are defined as a function of a common parameter) is very similar. For example, define the x coordinate as $x(t) = 3\,e^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)$ and the y coordinate $y(t) = \frac{d}{dt}x(t)$. Now plot $x(t)$ vs. $y(t)$ as t varies from 0 to 50.

Operation	**Clickable** 
Right-click on expression, and differentiate with respect to *t*. Copy and past both the function and derivative and place in new line separated by a comma. Mouse on the two functions, choose *Plots* → *Plot Builder*. In the dialog box, change the plot range to 0 to 50 (default is -10 to 10). Under *Select Plot* choose *2D Parametric Plot.*For the range, choose 0 to 50.	$3\,e^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)$ $3\,e^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)$ **(2.4.7.2.1)** differentiate w.r.t. t → $-\frac{e^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)}{2}+\frac{e^{-\frac{t}{6}}\cos\left(\frac{t}{6}\right)}{2}$ **(2.4.7.2.2)** $3\,e^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right), -\frac{e^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)}{2}+\frac{e^{-\frac{t}{6}}\cos\left(\frac{t}{6}\right)}{2}$ **(2.4.7.2.3)** →



Command version 

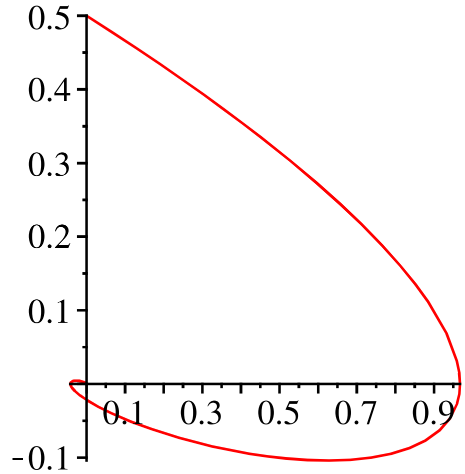
$expr := 3\,e^{-\frac{t}{6}}\sin\left(\frac{t}{6}\right)$

$$3\,e^{-\frac{1}{6}t}\sin\left(\frac{1}{6}t\right) \qquad \textbf{(2.4.7.2.1.1)}$$

$dexpr := diff(expr, t)$

$$-\frac{1}{2}\,e^{-\frac{1}{6}t}\sin\left(\frac{1}{6}t\right)+\frac{1}{2}\,e^{-\frac{1}{6}t}\cos\left(\frac{1}{6}t\right) \qquad \textbf{(2.4.7.2.1.2)}$$

$plot([expr, dexpr, t = 0..50])$

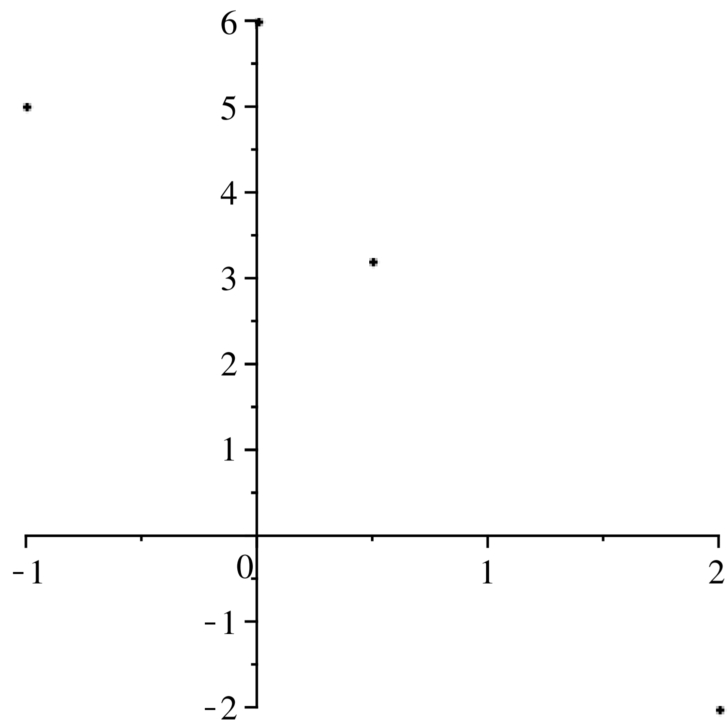


[Go back to Table of Contents](#)

2.4.7.3 Plotting Data

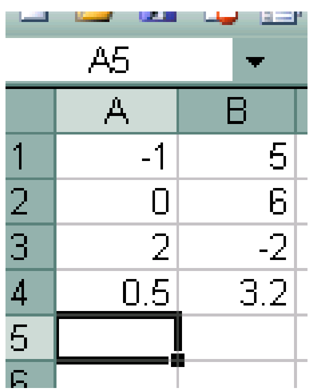
Maple can plot data from point-lists that you type in or from files. Maple is smart enough to interpret various data formats. Here are some very easy options.

Operation	**Clickable** 
Option 1: Enter the data yourself into Maple. Open the Matrix palette on the left. Press *Choose* and with the mouse pressed, drag and select 2 columns and as many rows as you have data points. Press *Insert Matrix.* Replace each element with your data. First column will be *x* values and second will be *y* values. The tab key will hop from one element to the next. Right click on finished matrix and select *Plots* → *Plot Builder*. 2D *Point plot* will be the only option available.	$\begin{bmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \\ m_{3,1} & m_{3,2} \\ m_{4,1} & m_{4,2} \end{bmatrix}$ will be displayed when you insert the matrix. Now highlight first element, make change then tab to next element. $\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ \frac{1}{2} & 3.2 \end{bmatrix} \rightarrow$



Option 2: Easy way of getting data in from Excel.

Enter your data into Excel with *x* values in one column and *y* values in the next column.



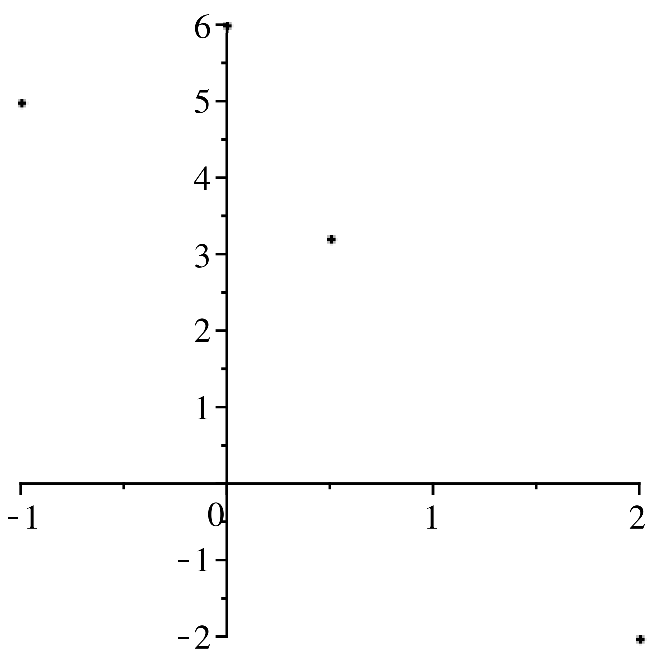
Highlight the desired data and *Copy*.

In Maple, *Paste* into a math area and follow the same procedure as above.

$$\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ 0.5 & 3.2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ 0.5 & 3.2 \end{bmatrix} \qquad \textbf{(2.4.7.3.1)}$$

→



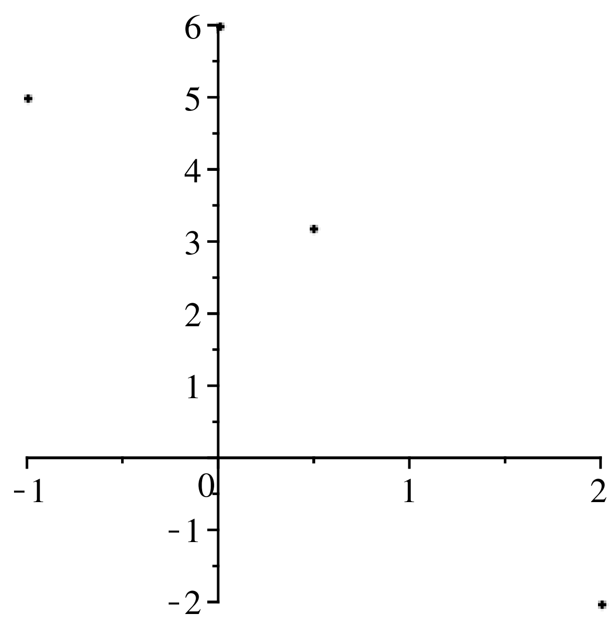
Command version

Although there are various Maple datastructures for matrices, it is still easiest to use the Matrix palette even when working with commands.

$$datamatrix := \begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ \frac{1}{2} & 3.2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ \frac{1}{2} & 3.2 \end{bmatrix} \quad \textbf{(2.4.7.3.1.1)}$$

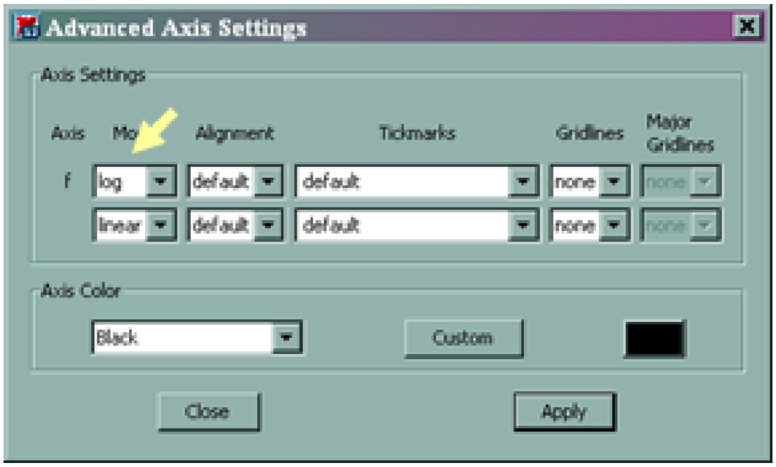
$plot(datamatrix, style = point, color = black)$



[*Go back to Table of Contents*]()

2.4.7.4 Logarithmic plots

Within the *Plot Builder*, you can specify logarithmic axes under *Options*, then *Axes*→*Advanced Options*, and choosing *Log* option.

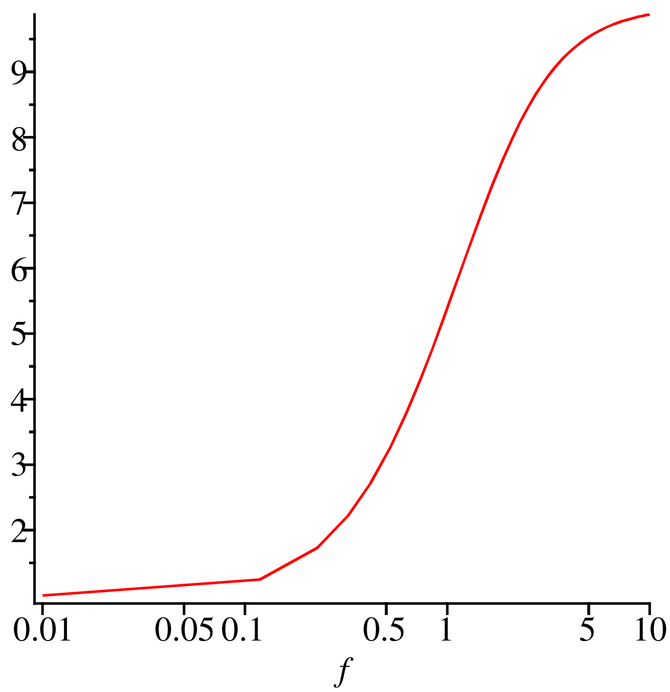


For example given the following, expression, the Plot Builder gives the gives the associated graph.

$$expr := \sqrt{\left(\frac{1}{1+0.04\,\pi^2 f^2}+\frac{0.4\,\pi^2 f^2}{1+0.04\,\pi^2 f^2}\right)^2+\frac{3.24\,\pi^2 f^2}{\left(1+0.04\,\pi^2 f^2\right)^2}}$$

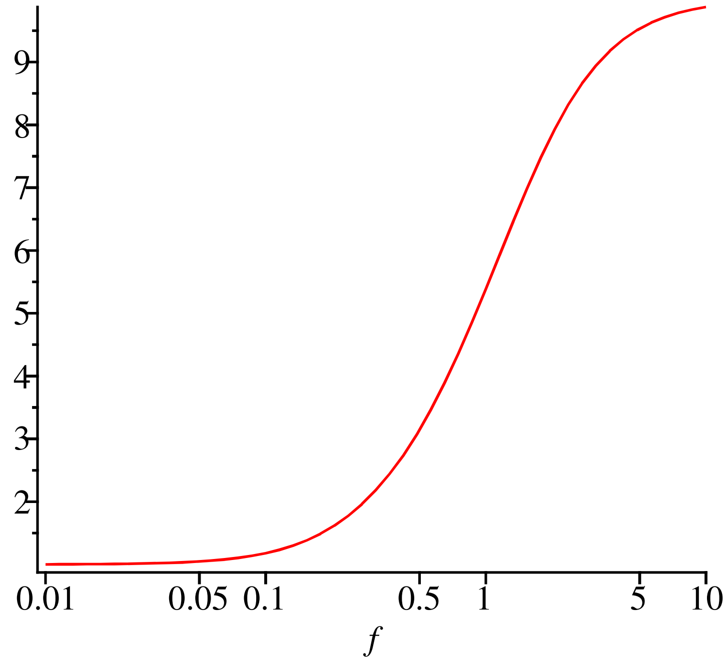
$$\sqrt{\left(\frac{1}{1+0.04\,\pi^2 f^2}+\frac{0.4\,\pi^2 f^2}{1+0.04\,\pi^2 f^2}\right)^2+\frac{3.24\,\pi^2 f^2}{\left(1+0.04\,\pi^2 f^2\right)^2}} \qquad (2.4.7.4.1)$$

→



Notice that for the lower values of *f*, the graph lacks detail and it appears to be undersampled. For many engineering applications requiring log axes, this is not quite good enough. The other, and better, option is to directly call the *semilogplot* call from the *plots* package. this routine has a smarter sampling routine that adds the appropriate levels of detail at the lower values.

$plots[semilogplot](expr, f = 0.01 .. 10)$



The *semilogplot* command has the log axis on the horizontal. The *logplot* command has the log scale on the vertical axis. The *loglogplot* command has the log plot on both axes.

Go back to Table of Contents

2.4.8 Basic differential and integral calculus

2.4.8.1 Limits, differentiation, integrals, series

Operation	**Clickable** 
Differentiation e.g. $\frac{d}{dx}(x^2 - x + 2)^{\frac{3}{4}}$: select from the *Expression Palette* [$\frac{d}{dx} f$]. Highlight the *f* and enter the expression. See Entering Maple expressions on how to do this. Press enter. Alternatively, you could just enter the expression, then right click on the first expression result and choose *Differentiate* → *x*.	$\frac{d}{dx}(x^2 - x + 2)^{\frac{3}{4}}$ $$\frac{3}{4} \frac{2x - 1}{(x^2 - x + 2)^{1/4}}$$ **(2.4.8.1.1)**
Indefinite integration e.g. $\int (x + \sec^2(\pi x))\, dx$: Select [$\int f\, dx$] and replace *f* with the expression. Press return. Alternatively, you could just enter the expression (the integrand) and choose *Integrate* → *x*. Note that Maple does not return a constant *C* as this constant is completely arbitrary and	$\int (x + \sec^2(\pi x))\, dx$ $$\frac{1}{2} x^2 + \frac{\sin(\pi x)}{\pi \cos(\pi x)}$$ **(2.4.8.1.2)**

Maple automatically makes the assumption that $C = 0$.	
Definite integration e.g. $\int_0^{\frac{\pi}{8}} \sin^5(2x)\cos(2x)\,dx$: Select [$\int_a^b f\,dx$] and make appropriate substitutions and press return. Note that there is no menu-based alternative. To get a floating point (decimal) representation of the answer, right-click and choose *Approximate → 5*.	$\int_0^{\frac{\pi}{8}} \sin^5(2x)\cos(2x)\,dx$ $\frac{1}{96}$ **(2.4.8.1.3)** $\xrightarrow{\text{at 5 digits}}$ 0.010417 **(2.4.8.1.4)**
Limits e.g. $\lim_{x\to\infty} \frac{3x+5}{6x-8}$: Select [$\lim_{x\to a} f$] and make appropriate substitutions and press return. You can get the ∞ from the *Common Symbols* palette. There is a possible menu-based through the *Constructions* option but this will generate additional steps and is much more cumbersome than using the palette.	$\lim_{x\to\infty} \frac{3x+5}{6x-8}$ $\frac{1}{2}$ **(2.4.8.1.5)**
Taylor series e.g. expand $\sqrt{1+x}$ to 8 terms about the point $x = 0$. There is no convenient clickable method. This example shows how you can construct the Taylor series using the fundamental definition, $\sum_{n=0}^{\infty} \frac{(x-a)^n f^{(n)}(a)}{n!}$ where $f^{(n)}$ is the n^{th} derivative with respect to x. Note that Maple also understands this notation. You will need to define a function f which can be conveniently differentiated and evaluated at the value $x = a$ in the numerator. The interpretation of the odd looking factor in the numerator is "evaluate the n^{th} derivative of the function $\sqrt{1+x}$ at $x = 0$.	$f := x \to \sqrt{1+x}$ $x \to \sqrt{x+1}$ **(2.4.8.1.6)** $\sum_{n=0}^{7} \frac{(x-0)^n \left(f^{(n)}(x) \Big\vert_{x=0} \right)}{n!}$ $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \frac{21}{1024}x^6 + \frac{33}{2048}x^7$ **(2.4.8.1.7)**

Command version 

Differentiation

$$diff\left((x^2 - x + 2)^{\frac{3}{4}}, x\right)$$

$$\frac{3}{4}\,\frac{2x-1}{(x^2-x+2)^{1/4}} \qquad \textbf{(2.4.8.1.1.1)}$$

Indefinite integration

$$int(x + \sec^2(\pi x), x)$$

$$\frac{1}{2}x^2+\frac{\sin(\pi x)}{\pi\cos(\pi x)} \quad \textbf{(2.4.8.1.1.2)}$$

Definite integration

$myint := int\left(\sin^5(2x)\cos(2x), x = 0..\frac{\pi}{8}\right)$

$$\frac{1}{96} \quad \textbf{(2.4.8.1.1.3)}$$

$evalf(myint, 5)$

$$0.010417 \quad \textbf{(2.4.8.1.1.4)}$$

Limits

$\lim\left(\frac{3x+5}{6x-8}, x = \infty\right)$

$$\lim\left(\frac{3x+5}{6x-8}, x = \infty\right) \quad \textbf{(2.4.8.1.1.5)}$$

Taylor Series

$taylor\left(\sqrt{1+x}, x = 0, 8\right)$

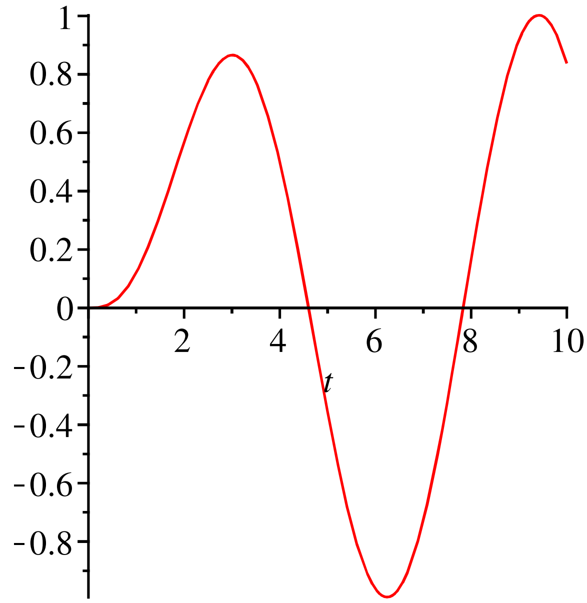
$$1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3-\frac{5}{128}x^4+\frac{7}{256}x^5-\frac{21}{1024}x^6+\frac{33}{2048}x^7+O(x^8) \quad \textbf{(2.4.8.1.1.6)}$$

Unlike the manual approach in the previous example, Maple automatically adds a $O(x^8)$ term indicating the order of the expansion.

Go back to Table of Contents

2.4.8.2 Ordinary differential equations

Operation	**Clickable** 
ODE with initial conditions e.g. $\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=\sin(t)$. First, enter the DE. For the second derivative, you need to use the palette symbol [$\frac{d}{dx}f$] twice in a nested fashion. Insert the symbol once. Change the *x* to a *t* then tab to highlight the *f*. Now, instead of typing something in, press on the [$\frac{d}{dx}f$] symbol again. Change the *x*, then replace the *f* with $y(t)$. **NOTE: It is very important that you specify the main variable as a function of** *t*: i.e. $y(t)$ instead of just *y*. Now, to solve the DE, right-click and choose *Solve DE* $\rightarrow y(t)$. A general solution is then returned. Note that the solution includes undetermined constants *_C1* and *_C2*. This is because we have not specified initial	$\frac{d}{dt}\left(\frac{d}{dt}y(t)\right)+\frac{d}{dt}y(t)+y(t)=\sin(t)$ $\frac{d^2}{dt^2}y(t)+\frac{d}{dt}y(t)+y(t)=\sin(t)$ **(2.4.8.2.1)** $\xrightarrow{\text{solve DE}}$ $y(t)=e^{-\frac{1}{2}t}\sin\left(\frac{1}{2}\sqrt{3}\,t\right)\_C2$ **(2.4.8.2.2)** $+e^{-\frac{1}{2}t}\cos\left(\frac{1}{2}\sqrt{3}\,t\right)\_C1-\cos(t)$

conditions.	
Try the same process again but this time, append the initial conditions (i.e. separated by a comma). Carefully note the notation for the derivative condition $D(y)(0)$. Make sure there are no spaces. Now when you solve, you get a more precise solution. Also note that the solution is returned as a mathematical equation. i.e. $y(t)$ = *solution expression*. To do anything with the solution, it is helpful to extract only the expression portion of the equation -- that is, the right hand side. Using the right-click menu, choose *Right-hand side*. Now it returns the expression. In this example, use the *Plot Builder* with t from 0 to 10 to see what the solution looks like.	$\frac{d}{dt}\left(\frac{d}{dt}y(t)\right)+\frac{d}{dt}y(t)+y(t)=\sin(t), y(0)=0, D(y)(0)$ $\xrightarrow{\text{solve DE}}$ $y(t)=\frac{1}{3}e^{-\frac{1}{2}t}\sin\left(\frac{1}{2}\sqrt{3}\,t\right)\sqrt{3}+e^{-\frac{1}{2}t}\cos\left(\frac{1}{2}\sqrt{3}\,t\right)-\cos(t)$ $\xrightarrow{\text{right hand side}}$ $\frac{1}{3}e^{-\frac{1}{2}t}\sin\left(\frac{1}{2}\sqrt{3}\,t\right)\sqrt{3}+e^{-\frac{1}{2}t}\cos\left(\frac{1}{2}\sqrt{3}\,t\right)-\cos(t)$ $\rightarrow$ 

Command version 

$de := diff(y(t), t, t) + diff(y(t), t) + y(t) = \sin(t)$

$$\frac{d^2}{dt^2}y(t)+\frac{d}{dt}y(t)+y(t)=\sin(t) \quad \textbf{(2.4.8.2.1.1)}$$

$dsolve(de)$

$$y(t)=e^{-\frac{1}{2}t}\sin\left(\frac{1}{2}\sqrt{3}\,t\right)\_C2+e^{-\frac{1}{2}t}\cos\left(\frac{1}{2}\sqrt{3}\,t\right)\_C1-\cos(t) \quad \textbf{(2.4.8.2.1.2)}$$

Now, with initial conditions

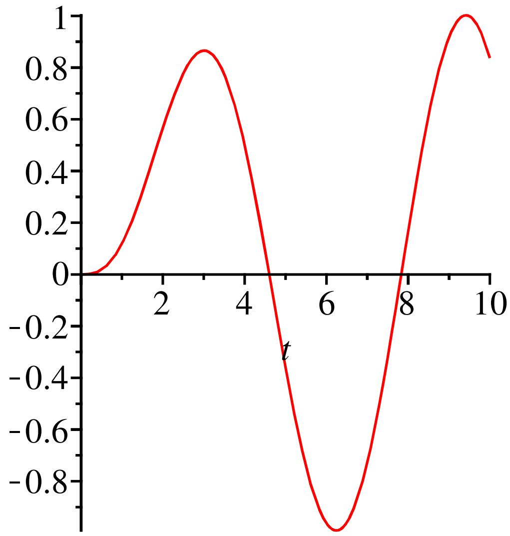
$sol := dsolve([de, y(0) = 0, D(y)(0)])$

$$y(t)=\frac{1}{3}e^{-\frac{1}{2}t}\sin\left(\frac{1}{2}\sqrt{3}\,t\right)\sqrt{3}+e^{-\frac{1}{2}t}\cos\left(\frac{1}{2}\sqrt{3}\,t\right)-\cos(t) \quad \textbf{(2.4.8.2.1.3)}$$

$solr := rhs(sol)$

$$\frac{1}{3} e^{-\frac{1}{2} t} \sin\left(\frac{1}{2} \sqrt{3}\, t\right) \sqrt{3} + e^{-\frac{1}{2} t} \cos\left(\frac{1}{2} \sqrt{3}\, t\right) - \cos(t) \quad \textbf{(2.4.8.2.1.4)}$$

plot(*solr*, *t* = 0 ..10)



[*Go back to Table of Contents*]

2.4.8.3 Systems of ODEs 

Consider two coupled second order ODEs. $\frac{dx_1^2}{dt^2} = x_2 - 2\,x_1$ and $\frac{dx_2^2}{dt^2} = x_1 - 2\,x_2$. First specify the differential equations.

$de1 := diff\left(x_1(t), t, t\right) = \left(x_2(t) - 2\,x_1(t)\right)$

$$\frac{d^2}{dt^2} x_1(t) = x_2(t) - 2\,x_1(t) \quad \textbf{(2.4.8.3.1)}$$

$de2 := diff\left(x_2(t), t, t\right) = \left(x_1(t) - 2\,x_2(t)\right)$

$$\frac{d^2}{dt^2} x_2(t) = x_1(t) - 2\,x_2(t) \quad \textbf{(2.4.8.3.2)}$$

Now collect the equations and the conditions into a list and use *dsolve* to solve. Maple returns a solution set with 2 distinct solutions $x_1(t)$ and $x_2(t)$. Assign the solution set to the variable *sol* so that it is easy to extract the desired solution and the expression.

$sol := dsolve\left(\left[de1, de2, x_1(0) = 1, x_2(0) = 0, D\left(x_1\right)(0) = 0, D\left(x_2\right)(0) = 0\right]\right)$

$$\left\{x_1(t) = \frac{1}{2}\cos\left(\sqrt{3}\, t\right) + \frac{1}{2}\cos(t), x_2(t) = -\frac{1}{2}\cos\left(\sqrt{3}\, t\right) + \frac{1}{2}\cos(t)\right\} \quad \textbf{(2.4.8.3.3)}$$

sol[1]

$$x_1(t) = \frac{1}{2}\cos\left(\sqrt{3}\, t\right) + \frac{1}{2}\cos(t) \quad \textbf{(2.4.8.3.4)}$$

rhs(*sol*[1])

$$\frac{1}{2}\cos\left(\sqrt{3}\, t\right) + \frac{1}{2}\cos(t) \quad \textbf{(2.4.8.3.5)}$$

[*Go back to Table of Contents*]

2.4.8.4 Numerical solutions to systems of ODEs 

Using the same example as above, solve the system numerically. As before, make the call to *dsolve* but this time, include the option *numeric* after the list of equations and conditions. Maple returns a "procedure object". Assign the name *sol* to this object.

$de1 := diff\left(x_1(t), t, t\right) = \left(x_2(t) - 2\,x_1(t)\right)$

$$\frac{d^2}{dt^2}\,x_1(t) = x_2(t) - 2\,x_1(t) \qquad \textbf{(2.4.8.4.1)}$$

$de2 := diff\left(x_2(t), t, t\right) = \left(x_1(t) - 2\,x_2(t)\right)$

$$\frac{d^2}{dt^2}\,x_2(t) = x_1(t) - 2\,x_2(t) \qquad \textbf{(2.4.8.4.2)}$$

$sol := dsolve\left(\left[de1, de2, x_1(0) = 1, x_2(0) = 0, \mathrm{D}\left(x_1\right)(0) = 0, \mathrm{D}\left(x_2\right)(0) = 0\right], numeric\right)$

$$\textbf{proc}\left(x\_rkf45\right)\ \ldots\ \textbf{end proc} \qquad \textbf{(2.4.8.4.3)}$$

This is basically a container that has all of the necessary information about the equations and structured in a way that it can conveniently calculate specific values or plots when needed. Try calculating the solution values when $t = 1$. You now get the specific values of your main variables and their first derivative.

$sol(1)$

$$\left[t = 1., x_1(t) = 0.189872875706317584, \frac{d}{dt}\,x_1(t) = -1.27552578790124894, x_2(t) = 0.350429433187948669, \frac{d}{dt}\,x_2(t) = 0.434054797638206258\right] \qquad \textbf{(2.4.8.4.4)}$$

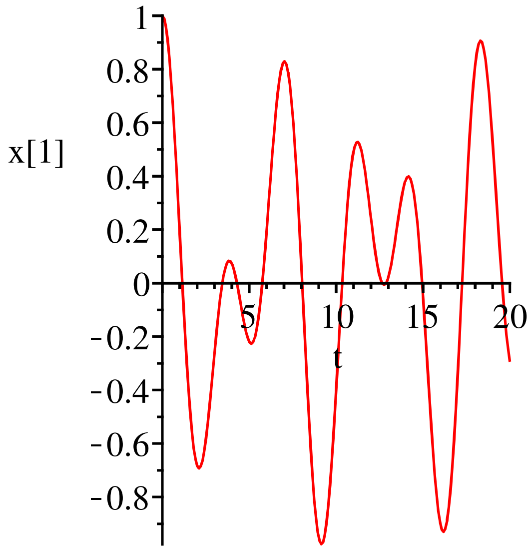
Typically, you would want to see the solution as a graph. There is a special command called *odeplot* which is contained in the *plots* package (a special collection of useful graphics tools). First load the package.

with(*plots*)

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *graphplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(2.4.8.4.5)**

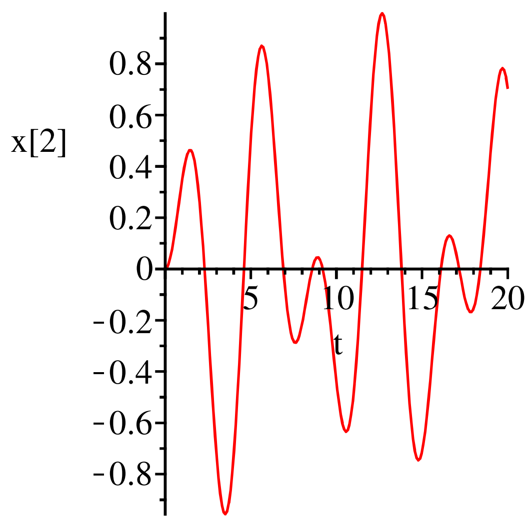
Now call *odeplot* and send in the name of the DE solution object. The *t* range you want to plot (i.e. 0 to 20 here), and to ensure a smooth plot, specify the number of calculation and plotting points to be 200. The default plot is the first state variable, in this case, $x_1(t)$.

odeplot(*sol*, 0 ..20, *numpoints* = 200)



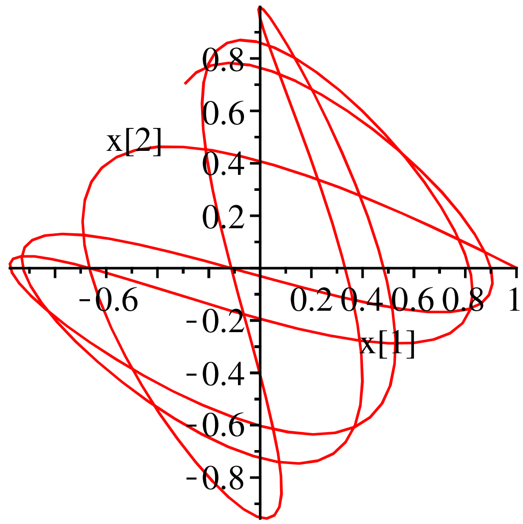
To view $x_2(t)$, enclose the horizontal axis variable *t* and $x_2(t)$, in a list. Note that you could have specified $x_1(t)$ in this way instead of taking the default in the above example step.

$odeplot(sol, [t, x_2(t)], 0..20, numpoints = 200)$



For many types of ODE systems, it is useful to plot one variable against the other. This is called a phase plane plot. To use *odeplot* to create a phase plane plot, place both variables in the list. Basically, $[x_1(t), x_2(t)]$ specifies a set of coordinates controlled by the parameter *t.* As *t* varies, it results in a different value for both x_1 and x_2 and consequently, this will carve out a pattern.

$odeplot(sol, [x_1(t), x_2(t)], 0..20, numpoints = 200)$



Go back to Table of Contents

2.4.9 Matrix operations

2.4.9.1 Defining a matrix or a vector

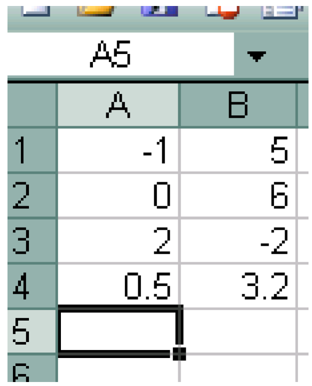
The easiest way to define a matrix or vector is with the Matrix palette. Using the *Choose* tool, you can conveniently specify arbitrary matrix or vector dimensions. You can even copy table of numbers from Excel and other packages and paste them as proper Maple matrices.

Operation	Clickable 
E.g. Define a 3×3 **matrix** Open the Matrix palette on the left. Press *Choose* and with the mouse pressed, drag and select 3 rows and 3 columns. Press *Insert Matrix.* Replace each element with your desired values. The tab key will hop from one element to the next. Press return	$\begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix}$ will be displayed when you insert the matrix. Now highlight first element, make change then tab to next element. $\begin{bmatrix} 1 & 5 & \frac{1}{2} \\ 2 & 0 & \omega_0^2 \\ -1 & 5 & 0.72 \end{bmatrix}$ $$\begin{bmatrix} 1 & 5 & \frac{1}{2} \\ 2 & 0 & \omega_0^2 \\ -1 & 5 & 0.72 \end{bmatrix} \quad \textbf{(2.4.9.1.1)}$$ Note that Maple accepts a wide range of data types for the elements.
E.g. Define a 3×1 **column vector** From Matrix palette, select 3 rows and	

<table>
<tr>
<td>one column and proceed as before.</td>
<td>$$\begin{bmatrix} 2 \\ -1 \\ \pi \end{bmatrix}$$
$$\begin{bmatrix} 2 \\ -1 \\ \pi \end{bmatrix} \quad \textbf{(2.4.9.1.2)}$$</td>
</tr>
<tr>
<td>E.g. Copy a matrix from Excel

Enter your data into Excel.

Highlight the desired data and Copy.

In Maple, Paste into a math area. The data will come in as a proper matrix.</td>
<td>

Now highlight, copy, and paste into Maple.
$$\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ 0.5 & 3.2 \end{bmatrix}$$
$$\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ 0.5 & 3.2 \end{bmatrix} \quad \textbf{(2.4.9.1.3)}$$</td>
</tr>
</table>

Command version 

Maple also has a low-level datastructure for matrices using a combination of "angle brackets" (i.e. less than "<" or greater than ">" symbols) and the vertical bar "|".

$$mymatrix := \left\langle \langle 1, 2, -1 \rangle \middle| \langle 5, 0, 5 \rangle \middle| \left\langle \frac{1}{2}, \omega_0^2, 0.72 \right\rangle \right\rangle$$

$$\begin{bmatrix} 1 & 5 & \frac{1}{2} \\ 2 & 0 & \omega_0^2 \\ -1 & 5 & 0.72 \end{bmatrix} \quad \textbf{(2.4.9.1.1.1)}$$

Caution: When using this notation, you enter the elements by columns (i.e. each grouping is a column). This is a bit tricky as the actual typing appears like a row.

$$mycolumnvector := \langle 2, -1, \pi \rangle$$

$$\begin{bmatrix} 2 \\ -1 \\ \pi \end{bmatrix} \tag{2.4.9.1.1.2}$$

Caution: again, typing a column vector with a command looks like you are defining a row vector. To type a row vector, do the following.

$myrowvector := \langle\langle 2\rangle|\langle -1\rangle|\langle \pi\rangle\rangle$

$$\begin{bmatrix} 2 & -1 & \pi \end{bmatrix} \tag{2.4.9.1.1.3}$$

[Go back to Table of Contents](#)

2.4.9.2 Matrix operations

2.4.9.2.1 Transpose, Determinant, Inverse

Operation	**Clickable**
Transpose a matrix Place cursor to the right of the matrix and raise the matrix to the power of "T" (i.e. the transpose notation). Use the "^" symbol to do this. or. Right click and choose *Standard operations* → *Transpose*	$\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ 0.5 & 3.2 \end{bmatrix}^T$ $\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ 0.5 & 3.2 \end{bmatrix}^T$ **(2.4.9.2.1.1)** or $\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ 0.5 & 3.2 \end{bmatrix} \xrightarrow{\text{transpose}} \begin{bmatrix} -1 & 0 & 2 & 0.5 \\ 5 & 6 & -2 & 3.2 \end{bmatrix}$
Inverse of a square matrix Raise matrix to the power of -1 or use right click menu and choose *Standard operations* → *Inverse*	$\begin{bmatrix} 1 & 5 & \frac{1}{2} \\ 2 & 0 & 2 \\ -1 & 5 & 7 \end{bmatrix}^{-1}$ **(2.4.9.2.1.2)**

	$$\begin{bmatrix} \frac{2}{17} & \frac{13}{34} & -\frac{2}{17} \\ \frac{16}{85} & -\frac{3}{34} & \frac{1}{85} \\ -\frac{2}{17} & \frac{2}{17} & \frac{2}{17} \end{bmatrix} \quad (2.4.9.2.1.2)$$ Try a singular matrix. The linear dependence will cause an error. $$\begin{bmatrix} 1 & 5 & \frac{1}{2} \\ 2 & 0 & 2 \\ 4 & 0 & 4 \end{bmatrix}^{-1}$$ `Error, (in rtable/Power) singular matrix`
Determinant of a square matrix Use the right click menu and choose *Standard operations* → *Determinant.* Note that Maple is capable of symbolic operations on matrices.	$$\begin{bmatrix} s+7 & 12 \\ -1 & s \end{bmatrix} \xrightarrow{\text{determinant}} s^2 + 7s + 12$$

Command version 

$$mymatrix := \begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ 0.5 & 3.2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 5 \\ 0 & 6 \\ 2 & -2 \\ 0.5 & 3.2 \end{bmatrix}$$

LinearAlgebra[*Transpose*](*mymatrix*)

$$\begin{bmatrix} -1 & 0 & 2 & 0.5 \\ 5 & 6 & -2 & 3.2 \end{bmatrix} \quad (2.4.9.2.1.1.2)$$

$$mymatrix := \begin{bmatrix} 1 & 5 & \frac{1}{2} \\ 2 & 0 & 2 \\ -1 & 5 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & \frac{1}{2} \\ 2 & 0 & 2 \\ -1 & 5 & 7 \end{bmatrix} \quad (2.4.9.2.1.1.3)$$

$LinearAlgebra[MatrixInverse](mymatrix)$

$$\begin{bmatrix} \frac{2}{17} & \frac{13}{34} & -\frac{2}{17} \\ \frac{16}{85} & -\frac{3}{34} & \frac{1}{85} \\ -\frac{2}{17} & \frac{2}{17} & \frac{2}{17} \end{bmatrix} \quad (2.4.9.2.1.1.4)$$

$$mymatrix := \begin{bmatrix} s+7 & 12 \\ -1 & s \end{bmatrix}$$

$$\begin{bmatrix} s+7 & 12 \\ -1 & s \end{bmatrix} \quad (2.4.9.2.1.1.5)$$

$LinearAlgebra[Determinant](mymatrix)$

$$s^2 + 7s + 12 \quad (2.4.9.2.1.1.6)$$

Go back to Table of Contents

2.4.9.2.2. Eigenvalues

The following examples compute the eigenvalues of a 5×5 matrix $\begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ -0.1 & -0.35 & 0.1 & 0.1 & 0.75 \\ 0 & 0 & 0 & 2 & 0 \\ 0.4 & 0.4 & -0.4 & -1.4 & 0 \\ 0 & -0.03 & 0 & 0 & -1 \end{bmatrix}$ in two different ways.

Operation	**Clickable** 
Right click menu *Eigenvalues, etc.* → *Eigenvalues.*	$\begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ -0.1 & -0.35 & 0.1 & 0.1 & 0.75 \\ 0 & 0 & 0 & 2 & 0 \\ 0.4 & 0.4 & -0.4 & -1.4 & 0 \\ 0 & -0.03 & 0 & 0 & -1 \end{bmatrix} \xrightarrow{\text{eigenvalues}}$ $\begin{bmatrix} -0.637107916517966388 + 0.666879401766211300\,I \\ -0.637107916517966388 - 0.666879401766211300\,I \\ 4.77821069300950660\;10^{-16} + 0.\,I \\ -0.507486110589103756 + 0.\,I \\ -0.968298056374965800 + 0.\,I \end{bmatrix}$ Note, the last three are real eigenvalues as the imaginary portion is 0. If you were to interpret this result, you would typically omit the $+0.I$. The next example shows an alternate method.
Compute the eigenvalues by calculating the roots of the characteristic equation.	

Choose *Characteristic Polynomial* from *Eigenvalue* menu. Right click on result and choose *Solve→Solve*	$\left[\begin{array}{ccccc} 0 & 2 & 0 & 0 & 0 \\ -0.1 & -0.35 & 0.1 & 0.1 & 0.75 \\ 0 & 0 & 0 & 2 & 0 \\ 0.4 & 0.4 & -0.4 & -1.4 & 0 \\ 0 & -0.03 & 0 & 0 & -1 \end{array}\right] \xrightarrow{\text{characteristic polynomial}}$ $1.88150\,\lambda^2 + 3.2225\,\lambda^3 + 2.75\,\lambda^4 + 0.41800\,\lambda + \lambda^5 \xrightarrow{\text{solve}} \{\lambda = 0.\},$ $\{\lambda = -0.6371079165 + 0.6668794018\,I\}, \{\lambda = -0.5074861106\}, \{\lambda = -0.9682980564\}, \{\lambda = -0.6371079165 - 0.6668794018\,I\}$

Command version 

$$mymatrix := \left[\begin{array}{ccccc} 0 & 2 & 0 & 0 & 0 \\ -0.1 & -0.35 & 0.1 & 0.1 & 0.75 \\ 0 & 0 & 0 & 2 & 0 \\ 0.4 & 0.4 & -0.4 & -1.4 & 0 \\ 0 & -0.03 & 0 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 0 & 2 & 0 & 0 & 0 \\ -0.1 & -0.35 & 0.1 & 0.1 & 0.75 \\ 0 & 0 & 0 & 2 & 0 \\ 0.4 & 0.4 & -0.4 & -1.4 & 0 \\ 0 & -0.03 & 0 & 0 & -1 \end{array}\right] \qquad (2.4.9.2.2.1.1)$$

$LinearAlgebra[Eigenvalues](mymatrix)$

$$\left[\begin{array}{c} -0.637107916517966388 + 0.666879401766211300\,I \\ -0.637107916517966388 - 0.666879401766211300\,I \\ 4.77821069300950660\,10^{-16} + 0.\,I \\ -0.507486110589103756 + 0.\,I \\ -0.968298056374965800 + 0.\,I \end{array}\right] \qquad (2.4.9.2.2.1.2)$$

$$mymatrix := \left[\begin{array}{ccccc} 0 & 2 & 0 & 0 & 0 \\ -0.1 & -0.35 & 0.1 & 0.1 & 0.75 \\ 0 & 0 & 0 & 2 & 0 \\ 0.4 & 0.4 & -0.4 & -1.4 & 0 \\ 0 & -0.03 & 0 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 0 & 2 & 0 & 0 & 0 \\ -0.1 & -0.35 & 0.1 & 0.1 & 0.75 \\ 0 & 0 & 0 & 2 & 0 \\ 0.4 & 0.4 & -0.4 & -1.4 & 0 \\ 0 & -0.03 & 0 & 0 & -1 \end{array}\right] \qquad (2.4.9.2.2.1.3)$$

$cpoly := LinearAlgebra[CharacteristicPolynomial](mymatrix, \lambda)$

$$1.88150\,\lambda^2 + 3.2225\,\lambda^3 + 2.75\,\lambda^4 + 0.41800\,\lambda + \lambda^5 \qquad (2.4.9.2.2.1.4)$$

$solve(cpoly)$

$$0., -0.6371079165 + 0.6668794018\,I, -0.5074861106, -0.9682980564, -0.6371079165 \qquad (2.4.9.2.2.1.5)$$

$-0.6668794018\,\mathrm{I}$

Go back to Table of Contents

2.4.9.2.3 Multiplication and matrix expressions

Operation	**Clickable** 
Scalar multiplication $s\,\mathbf{I}$ for a 2×2. If you are using the Matrix palette, you can either enter a generic matrix and replace the elements with 1s and 0s or you can explicitly enter an identity matrix with *Type* → *Identity*.	$s\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}$ **(2.4.9.2.3.1)**
Matrix multiplication To multiply a matrix and a vector, simply place the two objects side by side and Maple will properly interpret the multiplication and apply the proper rules. If you try the vector in the wrong form. Maple checks for dimension consistency and generates an error.	$\begin{bmatrix}0 & 1\\ \omega_0^2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ $\begin{bmatrix}x_2\\ \omega_0^2 x_1\end{bmatrix}$ **(2.4.9.2.3.2)** $\begin{bmatrix}0 & 1\\ \omega_0^2 & 0\end{bmatrix}\begin{bmatrix}x_2 & \omega_0^2 x_1\end{bmatrix}$ `Error, (in LinearAlgebra:-MatrixVectorMultiply) invalid input: LinearAlgebra:-MatrixVectorMultiply expects its 2nd argument, v, to be of type Vector[column] but received Vector[row](2, {(1) = x[2], (2) = omega[0]^2*x[1]})`
Complex example $\det\lvert\, s\mathbf{I} - (\mathbf{F} - \mathbf{G}\mathbf{K})\,\rvert.$ where **I** is the identity matrix, and **F**, **G**, and **K** correspond to those specified in the example on the right. Most of the computation can be done exactly as you would write the formula except for the determinant. This is done through the right-click menu, *Standard operations* →*Determinant.*	$s\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \left(\begin{bmatrix}0 & 1\\ \omega_0^2 & 0\end{bmatrix} - \begin{bmatrix}0\\ 1\end{bmatrix}\begin{bmatrix}K_1 & K_2\end{bmatrix}\right)$ $\begin{bmatrix}s & -1\\ -\omega_0^2 + K_1 & s + K_2\end{bmatrix}$ **(2.4.9.2.3.3)** $\xrightarrow{\text{determinant}}$ $s^2 + s K_2 - \omega_0^2 + K_1$ **(2.4.9.2.3.4)**

▼ *Command version* 

$$myidentity := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.1)}$$

$LinearAlgebra[ScalarMultiply](myidentity, s)$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.2)}$$

or

$s\, myidentity$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.3)}$$

$$mymatrix := \begin{bmatrix} 0 & 1 \\ \omega_0^2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ \omega_0^2 & 0 \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.4)}$$

$$myvector := \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.5)}$$

$LinearAlgebra[Multiply](mymatrix, myvector)$

$$\begin{bmatrix} x_2 \\ \omega_0^2 x_1 \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.6)}$$

You can also use the matrix "dot" operator (the "period" dot) which knows the special rules of matrix and vector multiplication. This should not be confused with the *dot product* (i.e. scalar product).

$mymatrix.myvector$

$$\begin{bmatrix} x_2 \\ \omega_0^2 x_1 \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.7)}$$

For multiplying 2 proper matrices, use the dot operator or the $LinearAlgebra[MatrixMultiply](M1, M2)$ command.

$$Fmatrix := \begin{bmatrix} 0 & 1 \\ \omega_0^2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ \omega_0^2 & 0 \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.8)}$$

$Gvector := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.9)}$$

$Kvector := \begin{bmatrix} K_1 & K_2 \end{bmatrix}$

$$\begin{bmatrix} K_1 & K_2 \end{bmatrix} \quad \textbf{(2.4.9.2.3.1.10)}$$

with(*LinearAlgebra*) :

Determinant(*Multiply*(*s*, *IdentityMatrix*(2)) − (*Fmatrix* − *Multiply*(*Gvector*, *Kvector*)))

$$s^2 + s K_2 - \omega_0^2 + K_1 \quad \textbf{(2.4.9.2.3.1.11)}$$

or

Determinant(*s*.*IdentityMatrix*(2) − (*Fmatrix* − *Gvector*.*Kvector*))

$$s^2 + s K_2 - \omega_0^2 + K_1 \quad \textbf{(2.4.9.2.3.1.12)}$$

Go back to Table of Contents

2.5 Dynamic systems package (control theory package)

Maple has a built-in package of various useful tools for control theory and linear systems concepts. It is called the *DynamicSystems* package and it can be loaded by typing in

with(*DynamicSystems*)

[*AlgEquation*, *BodePlot*, *CharacteristicPolynomial*, *Chirp*, *Coefficients*, *ControllabilityMatrix*, *Controllable*, *DiffEquation*, *DiscretePlot*, *FrequencyResponse*, *GainMargin*, *Grammians*, *ImpulseResponse*, *ImpulseResponsePlot*, *IsSystem*, *MagnitudePlot*, *NewSystem*, *ObservabilityMatrix*, *Observable*, *PhaseMargin*, *PhasePlot*, *PrintSystem*, *Ramp*, *ResponsePlot*, *RootContourPlot*, *RootLocusPlot*, *RouthTable*, *SSModelReduction*, *SSTransformation*, *Simulate*, *Sinc*, *Sine*, *Square*, *StateSpace*, *Step*, *System*, *SystemOptions*, *ToDiscrete*, *TransferFunction*, *Triangle*, *Verify*, *ZeroPoleGain*, *ZeroPolePlot*] **(2.5.1)**

The list of commands this returns are the various functions in the package. You can now refer to these commands simply through the command name. An alternative is the "long form" where you define the full function path. For example for the command *BodePlot,* the long form is

DynamicSystems[*BodePlot*]

If you refer to the command this way, you do not need to invoke the *with*(*DynamicSystems*) as before. The long form is useful if you want to do a quick one shot call and you want to guarantee that the function can be found by Maple. In these notes, you will see examples of both short and long form. The long form will always be in the form of,

Package name[*command name*]

The following are the long form name list of the most important commands in the *DynamicSystems* package. Names are hyperlinked to their respective help pages.

System definitions

- DynamicSystems[NewSystem] : create a system object compatible with the DynamicSystems package
- DynamicSystems[PrintSystem] : print the content of a system object

Responses and plots

- DynamicSystems[BodePlot] : plot magnitude and phase versus frequency

• DynamicSystems[ImpulseResponsePlot] : plot the impulse response of a system

• DynamicSystems[MagnitudePlot] : plot log magnitude versus frequency

• DynamicSystems[PhasePlot] : plot phase versus frequency

• DynamicSystems[ResponsePlot] : plot response of a system to a given input

• DynamicSystems[RootLocusPlot] : generate a root-locus plot

System calculations

• DynamicSystems[CharacteristicPolynomial] : compute the characteristic polynomial of a state-space system

• DynamicSystems[ControllabilityMatrix] : compute the controllability matrix

• DynamicSystems[GainMargin] : compute the gain-margin and phase-crossover frequency

• DynamicSystems[ObservabilityMatrix] : compute the observability matrix

• DynamicSystems[PhaseMargin] : return the phase-margin and gain-crossover frequency

• DynamicSystems[RouthTable] : generate the Routh table of a polynomial

Go back to Table of Contents

2.6 Simple programming in Maple

Most of this guide provides interactive (clickable) approaches to problem-solving. With most, you will also see the command equivalent. The command set that you use here is actually part of a complete programming language called the Maple language. In fact, if you read older references to Maple or speak to someone who has used the system for years, they often refer to Maple as a mathematical language rather than an integrated application.

Once you have become proficient in basic programming, you will be able to apply the underlying power of Maple to a wide variety of engineering and scientific problems. You can even use Maple to develop complete applications.

Go back to Table of Contents

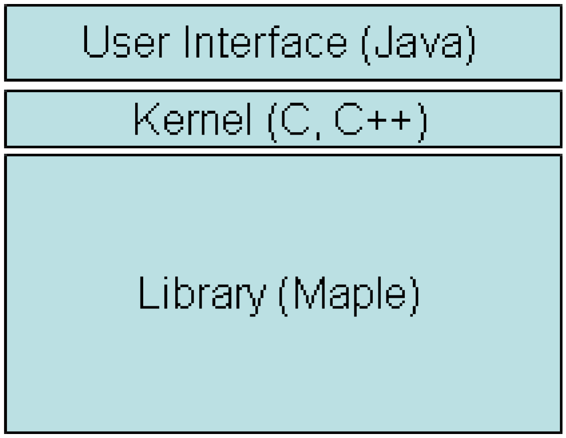
2.6.1 Basic architecture

The Maple system is organized in a way that math computations are very fast, easy to program, and can run on a wide range of different operating systems. The three most important parts of the system are,

• The Maple Kernel: as the "heart" of the system, this portion is relatively small in size and is written in C or C++. Because it is compiled, speed is very fast and consequently, all of Maple's low-level operations or anything that needs maximum speed is taken care of by the kernel. Additionally, the kernel understands the Maple programming language -- i.e. the kernel has a language "interpreter".

• The Maple Library: as the "brains" of the system, this portion is the largest portion and is written in the Maple language. The Maple language is optimized for math-oriented code so it takes care of many of the details that general languages such as Java or C require. As a result coding time is much shorter. Furthermore, because it is an interpreted language (as opposed to compiled), debugging and "quick and dirty" programs are much easier to write. When most users program in the Maple system, they are essentially talking about writing additional Maple code to supplement the built-in routines of the library.

• The User Interface: as the "face" of the system, this portion is written in the Java to facilitate the graphical interactions. In more recent versions (since Maple 10), it has been possible to customize the UI with relatively simple programming.



The fact that most of the math knowledge in Maple is in the Library and written in a user-friendly language (Maple), with a very modest programming effort, means that you can perform computing tasks that would take days to program in a conventional language.

Go back to Table of Contents

2.6.2 Programming approach

Programming in Maple can mean anything from writing a group of command lines and executing them. The various examples in this guide that use commands instead of clickable approaches are intended to be executed one command at a time: i.e. type a command, press enter, see result, repeat for the next command, repeat etc. The following example shows how you can develop this basic idea to more useful reusable tools using some very simple Maple techniques.

2.6.2.1 First step: solve problem interactively

For example, consider the following steps for the computation of the characteristic equation for a closed loop transfer function.

$G := \frac{1}{s^2+2s-1}$

$$\frac{1}{s^2+2s-1} \qquad (2.6.2.1.1)$$

$H := \frac{1}{s+1}$

$$\frac{1}{s+1} \qquad (2.6.2.1.2)$$

$TF := normal\left(\frac{G H}{1+G H}\right)$

$$\frac{1}{s\,(s^2+3s+1)} \qquad (2.6.2.1.3)$$

$chareq := denom(TF)$

$$s\,(s^2+3s+1) \qquad (2.6.2.1.4)$$

Go back to Table of Contents

2.6.2.2 Variation 1: enter same commands in a single block

For this next step, let's re-enter the above commands but this time **instead of pressing [Enter] with each line, you press [Shift][Enter]** simultaneously. Doing this will suppress execution and let you enter a block of commands. Also, **insert a semicolon ";" after each command including the last command**. This logically separates the lines so that

Maple knows they are distinct commands and not something that is a continuation. Once complete, press enter and all of the commands will then execute.

$G := \frac{1}{s^2 + 2\,s - 1};$
$H := \frac{1}{s+1};$
$TF := normal\left(\frac{G\,H}{1 + G\,H}\right);$
$chareq := denom(TF);$

$$\frac{1}{s^2 + 2\,s - 1}$$
$$\frac{1}{s+1}$$
$$\frac{1}{s\,(s^2 + 3\,s + 1)}$$
$$s\,(s^2 + 3\,s + 1) \quad \textbf{(2.6.2.2.1)}$$

The output shows the result of each step and therefore, the last line of the output group is the characteristic equation.

The above is a very simple Maple "program". If you simply went back and edited G or H for other functions, with a simple press of the [Enter] key you can get the new characteristic equation. Also note that the variable *chareq* now contains the characteristic equation and can be used in other calculations.

Go back to Table of Contents

2.6.2.3 Variation 2: compose a function

The previous simple "program" can be improved by using the function or operator notation that was introduced in Defining Functions $f(x)$.

$getchareq := (G, H) \rightarrow denom\left(normal\left(\frac{G\,H}{1 + G\,H}\right)\right)$

$$(G, H) \rightarrow denom\left(normal\left(\frac{G\,H}{1 + G\,H}\right)\right) \quad \textbf{(2.6.2.3.1)}$$

$getchareq\left(\frac{1}{s^2 + 2\,s - 1}, \frac{1}{s+1}\right)$

$$s\,(s^2 + 3\,s + 1) \quad \textbf{(2.6.2.3.2)}$$

$getchareq\left(\frac{s+1}{s^2 + s - 1}, K_p\right)$

$$s^2 + s - 1 + K_p\,s + K_p \quad \textbf{(2.6.2.3.3)}$$

This is a much more elegant and convenient form. Now you can simply call this new function with a single new command. The problem with this approach is, of course, if the steps become greater in number or complexity, the single-line functional programming becomes a real challenge.

Go back to Table of Contents

2.6.2.4 Variation 3: create a procedure

The most popular form for a Maple program is typically in the form of a *procedure* which is the official name of Maple's subroutine. For example enter the following **using [Shift][Enter] for each line**,

$getchareqproc := \mathbf{proc}(G, H)$
$TF := normal\left(\frac{G H}{1 + G H}\right);$
$denom(TF);$
end proc;

```
Warning, `TF` is implicitly declared local to procedure
`getchareqproc`
```

$\mathbf{proc}(G, H)$ **local** TF; $TF := normal(G * H / (1 + G * H))$; $denom(TF)$ **end proc** **(2.6.2.4.1)**

$getchareqproc\left(\frac{1}{s^2 + 2 s - 1}, \frac{1}{s + 1}\right)$

$$s (s^2 + 3 s + 1) \quad \textbf{(2.6.2.4.2)}$$

The result returned by a procedure is generally the output of the last command executed -- in this case, the command $denom(TF)$. Also note that it warns of the local nature of the variable *TF*. Maple procedures assume all subroutine variables are local unless specified otherwise. The procedure is very similar in structure and style with subroutines from other languages. More experienced programmers can add more sophistication simply by adding lines of code.

Go back to Table of Contents

2.6.2.5 Variation 4: loops and conditions

Let's consider a more complex procedure that will accommodate an arbitrary number of functions and use the formula $\frac{G_1 G_2 ... G_n}{1 + G_1 G_2 ... G_n}$ to compute the transfer function and check for the degree of the characteristic equation and print a message if the degree is greater than 3 (why do you think it is important for us to know is a degree of a polynomial is greater than 3?).

$getchareqproc2 := \mathbf{proc}(Glist, n)$

local *Gprod*, *i*, *TF*, *ceq*, *ceqdeg*;

 # set product variable to 1. Loop through all functions and compute product
$Gprod := 1;$
for *i* **from** 1 **to** *n* **do**
$Gprod := Gprod \cdot Glist[i];$
end do;

 # get characteristic equation
$TF := normal\left(\frac{Gprod}{1 + Gprod}\right);$
$ceq := denom(TF);$

 # get the degree of the characterisic equation
$ceqdeg := degree(ceq);$

 # check the degree. If > 3 return short message, the degree, the equation, otherwise, return just the equation
if $(ceqdeg > 3)$ **then**
"Attention ... degree is", *ceqdeg*, *ceq*
else
ceq;
end if;

end proc;

$\mathbf{proc}(Glist, n)$ **(2.6.2.5.1)**
local *Gprod*, *i*, *TF*, *ceq*, *ceqdeg*;
$Gprod := 1;$

```
    for i to n do Gprod := Gprod * Glist[i] end do;
    TF := normal(Gprod / (Gprod + 1));
    ceq := denom(TF);
    ceqdeg := degree(ceq);
    if 3 < ceqdeg then "Attention ... degree is", ceqdeg, ceq else ceq end if
end proc
```

$getchareqproc2\left(\left[\frac{1}{s}, \frac{1}{s-1}, \frac{1}{3}, \frac{2}{s+2}\right], 4\right)$

$$2 + 3\,s^3 + 3\,s^2 - 6\,s \qquad \textbf{(2.6.2.5.2)}$$

$getchareqproc2\left(\left[\frac{1}{s}, \frac{1}{s^2+s-1}, \frac{K}{s}\right], 3\right)$

$$\text{"Attention ... degree is"}, 4, K + s^4 + s^3 - s^2 \qquad \textbf{(2.6.2.5.3)}$$

Notes on the above enhanced procedure:

The syntax for the *for* loop is
for *index variable* **from** *start value* **to** *final value* **do**
    steps of the loop ...
end do;

The syntax for the conditional branch is,
if (*condition test*) **then**
    first possible result
else
    other possible result
end if;

The functions were collected as single list (i.e. a group contained in square brackets) that was named *Glist* within the procedure. This way, the procedure has a single name for the whole collection and can simply use an index to access individual functions.

All local variables were declared as such.

The Maple comment character is #

The output line "Attention ... degree is", *ceqdeg*, *ceq* within the *if* structure is a crude way of stringing together the pieces for the message. There are, of course, more polished ways of printing messages and returning results.

Go back to Table of Contents

2.6.2.6 Variation 5: error trapping

This version introduces a very convenient command *nops* which returns the "number of operands". So in this case, the procedure uses *nops* to count the number of function in the incoming list instead of requiring the user to explicitly specify the number.

It also introduces the *error* command which, when used with the *if*, gives a convenient way to introduce error trapping and friendly messages. In this case, the procedure checks for a valid number of functions within the list and exits the procedure if the user has left the list blank.

```
getchareqproc3 := proc(Glist)

 local n, Gprod, i, TF, ceq, ceqdeg;

 # get number of functions in the list

   n := nops(Glist);
   if n < 1 then error "Please enter at least one function" end if;
```

```
# set product variable to 1. Loop through all functions and compute product
  Gprod := 1;
  for i from 1 to n do
    Gprod := Gprod·Glist[i];
  end do;

# get characteristic equation
  TF := normal(Gprod/(1 + Gprod));
  ceq := denom(TF);

# get the degree of the characterisic equation
  ceqdeg := degree(ceq);

# check the degree. If > 3 return short message, the degree, the equation, otherwise, return just the equation
  if (ceqdeg > 3) then
    "Attention ... degree is", ceqdeg, ceq
  else
    ceq;
  end if;

end proc;
```

```
proc(Glist)
    local n, Gprod, i, TF, ceq, ceqdeg;
    n := nops(Glist);
    if n < 1 then error "Please enter at least one function" end if;
    Gprod := 1;
    for i to n do Gprod := Gprod * Glist[i] end do;
    TF := normal(Gprod / (1 + Gprod));
    ceq := denom(TF);
    ceqdeg := degree(ceq);
    if 3 < ceqdeg then "Attention ... degree is", ceqdeg, ceq else ceq end if
end proc
```
(2.6.2.6.1)

$getchareqproc3\left(\left[\frac{1}{s}, \frac{1}{s-1}, \frac{1}{3}, \frac{2}{s+2}\right]\right)$

$$3\,s^3 + 3\,s^2 - 6\,s + 2 \quad \textbf{(2.6.2.6.2)}$$

$getchareqproc3([\ ])$

```
Error, (in getchareqproc3) Please enter at least one function
```

Go back to Table of Contents

2.7 Maple Gotchas -- DON'T FORGET!

Maple is used by hundreds of thousands of people and many different fields. As such, it has aspects that may make perfect sense for one type of user but seem very odd to another. The following are some fundamental elements of Maple that many new users often trip up with. For now, simply familiarize yourself with these tips and tricks and you will encounter many fewer issues as you develop your Maple proficiency.

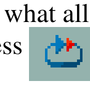
RULE NUMBER ONE: Maple is not human!

Yes it may seem intelligent because it can produce answers to math problems that would take you days to solve but remember that Maple is still basically software that follows digital recipes and algorithms to produce the answers. In some cases, it cannot find an answer to a problem but perhaps with your human insight you can immediately deduce it; or maybe it orders the terms of a polynomial differently than you normally would. In either case, it is not the most productive use of your time to fight with the software so that it is visually more to your liking or trying to force it to deal with mathematical operations that it has not been programmed to do (yet). Once you accept the deterministic and finite nature of this software, you will quickly identify all of good applications for it and find quick and easy ways to work around the rough parts.

Maple is case sensitive

Maple cares about the difference between *myvariable* and *Myvariable*. When following the examples in this guide make sure you replicate the steps faithfully.

Don't forget to Restart

If you are doing a lot of little tasks in a given Maple session (for example, working through various examples in these notes), you should get in the habit of forcing a memory-clear or "restart" which clears all of your variable definitions. For instance, if you defined a variable a to have some value like $\frac{1}{3}$ in a previous calculation and later you want to specify a general expression $a^2 + 1$ but you want a to be an unknown, you may end up in a state where you are not quite sure what all the values are. So, if you are at the very beginning of a problem, clearing memory is a good idea. To restart, either press  in the main toolbar or if you are using commands, type *restart.*

Assignments ($:=$) vs. equalities (=)

Assignments are used to give variable names to expressions. So, $f := \sin(x)$ reads as, "let the variable name f represent the expression $\sin(x)$. The equal sign without the colon is considered a statement mathematical equality. So, $f = \sin(x)$ reads as "the mathematical quantity f is equal to $\sin(x)$. The equal sign is typically used in logical comparisons ("if then else" or other Boolean comparisons -- equivalent to the double equal operator == in the C language) or in composing actual mathematical equations. In Maple, it is valid to state $f := \sin(x) = \frac{1}{y}$ which reads "let the variable name f represent the implicit equation $\sin(x) = \frac{1}{y}$.

Variables (f) vs. functions ($f(x)$)

In Maple, if you define a standard variable (e.g. $f := \sin(x)$), you cannot immediately use that variable name for function evaluations $f(x)$ (e.g. to calculate $\sin(2.5)$, $\sin\left(\frac{\pi}{2}\right)$, etc.). Maple does provide several convenient ways to work with functions. Examples are shown in the section Defining Functions.

Showing multiplications

In the regular formatted math display mode, multiplication can be specified in one of two ways:
- explicitly type a * character (star). The expression will then show a centered dot to denote multiplication (e.g. typing 2*k/3 will give $\frac{2 \cdot k}{3}$).
- use a space to imply multiplication. This is closer to what you would typically do if you were to write mathematical expressions by hand. In this case, no center dot will appear and multiplication is implied by the juxtaposition (e.g. type 2 [space] k/3 will give $\frac{2\,k}{3}$).

The first option is unambiguous and is preferred by many users. This guide tends to use the second way because math becomes more intuitive for reading and once you get used to it, it is the fastest way to enter formulas. At the outset though, you should take extra care in "seeing" the spaces. For example $k\,x$ (i.e. k [space] x) is k multiplied by x. If you omit the space, you will get kx which Maple interprets as a single variable with the name kx. Also, if you have $x(t)$, Maple will interpret this as "x as a function of t" and not "x times t". Conversely, if you include a space in a function name (e.g. f [space] (t) or $f\ (t)$), Maple will not see a function but f times t.

This may seem complicated but you get used to it very quickly. If you find yourself not getting used to it soon enough, feel free to use the * notation and any of the examples.

Number formats: fractions vs. decimal numbers

Maple, by default, expresses numerical answers in terms of fractions and irrational numbers (e.g. π, $\sqrt{2}$, etc.). This ensures that no roundoff errors are generated (e.g. $\frac{1}{3}$ is exact but the floating point equivalent 0.33333 ... will always be an approximation of $\frac{1}{3}$ regardless of the number of digits you carry. In Maple, you can conveniently switch from exact to floating point. Details are in the section Numerical Formats.

Indexed or subscripted variables

To some, the variables K_p, K_i, and K are three different variables. To Maple, this is actually tricky situation. Two of the three have subscripts or an index while the last is just plain K. This can cause confusion as subscripted variable in Maple behave differently than unsubscripted variables. The preferred situation for Maple is if all were subscripted or none were subscripted. For example using variables K_p, K_i, and K_0 is fine or removing the subscripts altogether and using *Kp*, *Ki*, and *K* are both fine.

Semicolons and colons

Older versions of Maple (i.e. before Maple 10) enforced either a semicolon (;) or colon(:) at the end of every line if you are using commands. With newer versions, when you are working in the normal "document" mode, there is no need. The only exception is when you program and you need to group lines of code together, the semicolon or colon are critical. Most exercises in this guide omit these characters with the exception of those illustrating programming techniques. If you are using these characters, the semicolon assumes you want to see the output and the colon suppresses output.

Packages and the with command

Not all Maple commands are actually available to you at startup. There is a fairly important and large collections of commands contained in something called a Maple "package". Examples of important packages are *LinearAlgebra* (linear algebra and matrix operations), *inttrans* (integral transforms including Laplace transforms), *plots* (special plotting routines), and *DynamicSystems* (special functions for engineering modeling including control systems). They are included with the software but must be "loaded" into the Maple session. There are two common ways to do this.

The "long form" combines both the package name and the individual command in the form *package name*[*command*] (no space). For example *inttrans*[*laplace*] is the long form for the Laplace transform command. You would need to specify the long form every time you access the command.

The second way is using the *with* command in the form *with*(*package name*) before you execute any of the commands. This approach loads all of the commands at once and from that point on you can access the command by the command name only. For example, executing *with*(*inttrans*) first will allow you to do the Laplace transform simply with the command *laplace*.

The benefit of the long form is that you are always sure of getting the command. The *with* approach saves on typing. Ultimately, it does come down to personal preference.

Go back to Table of Contents

2.7.1 Special Gotcha's for Control Systems

Maple has a few elements that often cause problems if you are doing work relating to control systems or signal processing. The following are the most prominent.

Complex numbers: *i* **vs**. *j*

By default, Maple calls $\sqrt{-1}$ as *i*. This is traditionally the letter used in mathematics. Within most of the engineering world, the preferred letter for the imaginary number is *j* as many want *i* reserved typically for electrical current. Furthermore, if you are typing out Maple commands that require *i* (i.e. *j*), you need to type *I* (capital *i*). Although there are techniques to switch definitions within Maple, the safest approach is still to use *I*.

ζ (**damping coefficient vs. the Riemann "zeta" function**)

The ζ (Greek letter zeta) is used in virtually all control or engineering modeling contexts as a damping coefficient. Within Maple, ζ is reserved for something called the Riemann Zeta function. If you ever include a ζ into a Maple expression, chances are Maple will gladly accept the variable but give you totally incorrect results. Again there are ways to change Maple's behavior but that can cause uncertainty in your computations. The recommended option is to use the Greek letter ξ (xi) instead of ζ for the damping coefficient -- i.e. a "double squiggle" instead of "single squiggle". You will find that the look is very similar and there is no chance of conflict. The examples in these notes use the ξ.

Go back to Table of Contents

2.7.2 Formats and modes

Maple offers two basic styles of interaction: "Document" mode and "Worksheet" mode. Roughly speaking, Document mode is the modern Maple environment that allows greater levels of interaction and sophisticated mathematical word processing features. Conversely, the Worksheet mode is the traditional Maple environment where you typically type in a command, press [Enter], and Maple responds with the answer. This mode is still popular among those who program or those wishing to use older Maple worksheets from versions of Maple prior to Maple 10.

The newer versions of Maple (since Maple 10) supports both of these modes. This guide assumes that you are principally using the Document mode. This approach offers the nice benefit of all of the new "clickable", user-friendly options and when needed, you can still do modest amounts of programming.

The following is a brief summary of the various modes and mode options that you may wish to explore in the future.

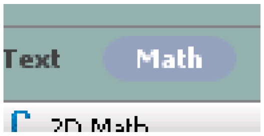
The document mode lets you mix calculations, text, tables, graphics, and more in a single flexible environment. You typically enter math in what is referred to as 2D mode -- i.e. the math is laid out naturally. When you use this mode, your math information is easier to read and you have access to more interactive options. This is the mode that we use in these notes. When you first install Maple, you will be asked whether you want the default to be a *Document* or *Worksheet*. For the purposes of these notes, you should choose Document. In the *File* menu, under *New*, you can choose to create a new Document or Worksheet at any time. so you will not be stuck in the future with either choice.

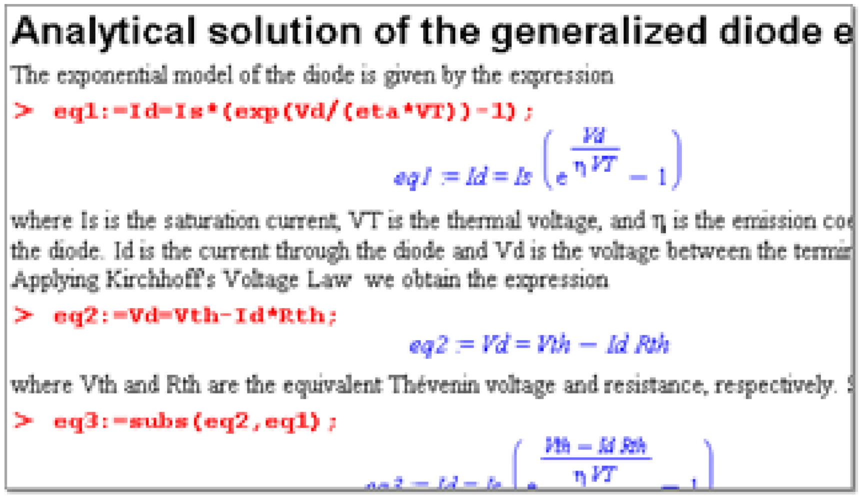
Enforcing floating point calculations

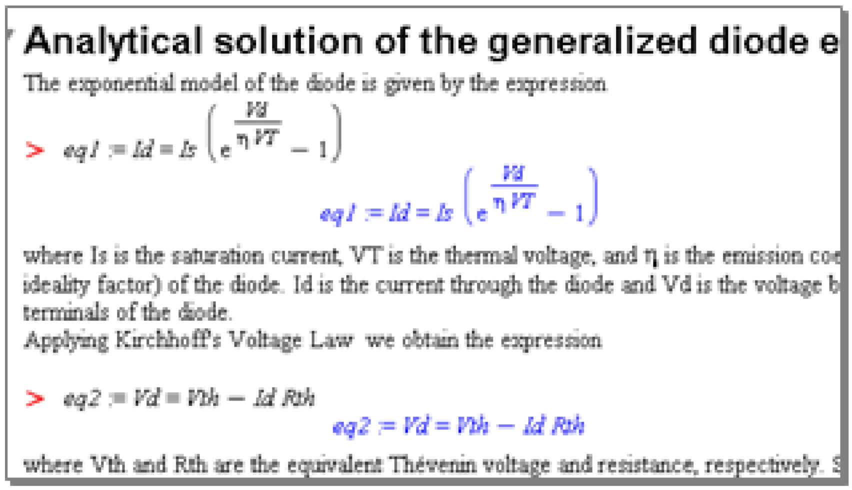
If you prefer to always see the floating point versions of expressions, you can exp
floating point calculation from that point on.

Operation	Clickable
For the expression, replace numbers with decimal equivalents	$\dfrac{3.\, e^{-\frac{t}{6.}} \sin\left(\dfrac{\sqrt{35.}\, t}{6.}\right) \sqrt{35.}}{35.}$ $0.5070925528\, e^{-0.1666666667\, t} \sin(0.9860132974\, t)$ **(2.1.7.2.1)**

The Worksheet mode is Maple's traditional environment that allows for basic command input and output and limited word-processing. In Worksheet mode, depending on your settings, you can either enter your commands in the linear "1D" or "red" form or the 2D form (as in the Document mode). The benefit of the first is that you can see exactly what you typed and the benefit of the second is that it is easier to read complicated math. You can switch between the 1D and 2D form with [F5] or the Text/Math toggle button on the toolbar -- i.e. in Worksheet mode, "Text" means 1D math, and "Math" means 2D math, somewhat analogous to the behavior of these toggles in document mode (i.e. regular text vs. executable math).







[Go back to Table of Contents]

3. Mathematical foundations

3.1 Complex variables

Operation	Clickable 
Imaginary and complex numbers. $\sqrt{-1}$ is represented by the letter I (note that it is always capital), and not j or i. A complex number has the general form $a + b\,I$.	$\sqrt{-1}$ I **(3.1.1)** $1 + 2\,I$ $1 + 2\,\mathrm{I}$ **(3.1.2)**
Complex arithmetic	$A := 1 + 2\,I$ $1 + 2\,\mathrm{I}$ **(3.1.3)** $B := -3 - 4\,I$ $-3 - 4\,\mathrm{I}$ **(3.1.4)** $A + B$ $-2 - 2\,\mathrm{I}$ **(3.1.5)** $A\,B$ $5 - 10\,\mathrm{I}$ **(3.1.6)** $\frac{A}{B}$ $-\frac{11}{25} - \frac{2\,\mathrm{I}}{25}$ **(3.1.7)** A^2 $-3 + 4\,\mathrm{I}$ **(3.1.8)**
Magnitude and phase are also referred to as the absolute value and argument. Use the right click menu for the clickable operations. In the expression palette there is also a symbol $\lvert a \rvert$ which you can also use for the absolute value.	$1 + 2\,I$ $1 + 2\,\mathrm{I}$ **(3.1.9)** $\xrightarrow{\text{absolute value}}$ $\sqrt{5}$ $1 + 2\,I$ $1 + 2\,\mathrm{I}$ **(3.1.10)** $\xrightarrow{\text{complex argument}}$ $\arctan(2)$ **(3.1.11)** $\lvert 1 + 2\,I \rvert$ $\sqrt{5}$ **(3.1.12)**

Command version 

$A := 1 + 2\,I$

$$1 + 2\,\mathrm{I} \quad \textbf{(3.1.1.1)}$$

abs(A)

$$\sqrt{5} \quad \textbf{(3.1.1.2)}$$

argument(A)

$$\arctan(2) \quad \textbf{(3.1.1.3)}$$

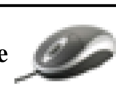
Go back to Table of Contents

3.2 Laplace transforms

The following are examples of computing Laplace transforms.

Operation	Clickable 
$\mathcal{L}\{\sin(10\,t)\}$	$\sin(10\,t)$ $\sin(10\,t)$ **(3.2.1)** $\xrightarrow{\text{Laplace transform}}$ $\frac{10}{s^2+100}$ **(3.2.2)**
Unit step and impulse functions (Heaviside step and Dirac delta functions). Note: Maple's default notation for the step function is $\theta(t)$. From right-click menu, choose *Integral Transform → Laplace*. Choose t for transform variable and s for transform parameter.	$\text{Heaviside}(t-1)$ $\text{Heaviside}(t-1)$ **(3.2.3)** $\xrightarrow{\text{Laplace transform}}$ $\frac{e^{-s}}{s}$ $\delta(t-1)$ $\text{Dirac}(t-1)$ **(3.2.4)** $\xrightarrow{\text{Laplace transform}}$ e^{-s} **(3.2.5)**
Derivatives $\mathcal{L}\left\{\frac{d}{dt}y(t)\right\}$ The result produces a placeholder for the transformed function $Y(s)$	$\frac{d}{dt}\,y(t)$ $\frac{1}{(s+1)^m}$ **(3.2.6)** $\xrightarrow{\text{Laplace transform}}$ $s\,laplace(y(t),t,s)-y(0)$ **(3.2.7)** In books, you would typically see this result expressed as, $s\,Y(s)-y(0)$.

The following are examples of computing inverse Laplace transforms.

Operation	Clickable 
$\mathcal{L}^{-1}\left\{\frac{10}{s^2+100}\right\}$ From the right-click menu, choose *Integral Transform → Inverse Laplace.*	$\frac{10}{s^2+100}$ $\frac{10}{s^2+100}$ **(3.2.8)** $\xrightarrow{\text{inverse Laplace transform}}$ $\sin(10\,t)$

$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s}\right\}$	$\frac{e^{-s}}{s}$ $\frac{e^{-s}}{s}$ **(3.2.9)** $\xrightarrow{\text{inverse Laplace transform}}$ Heaviside$(t-1)$

Command version

$inttrans[laplace](\sin(10\,t), t, s)$

$$\frac{10}{s^2+100} \quad \textbf{(3.2.1.1)}$$

$inttrans[laplace](\text{Heaviside}(t-1), t, s)$

$$\frac{e^{-s}}{s} \quad \textbf{(3.2.1.2)}$$

$inttrans[laplace](\text{Dirac}(t-1), t, s)$

$$e^{-s} \quad \textbf{(3.2.1.3)}$$

$inttrans[laplace](diff(y(t), t), t, s)$

$$s\,laplace(y(t), t, s) - y(0) \quad \textbf{(3.2.1.4)}$$

$inttrans[invlaplace]\left(\frac{10}{s^2+100}, s, t\right)$

$$\sin(10\,t) \quad \textbf{(3.2.1.5)}$$

$inttrans[invlaplace]\left(\frac{e^{-s}}{s}, s, t\right)$

$$\text{Heaviside}(t-1) \quad \textbf{(3.2.1.6)}$$

Go back to Table of Contents

3.3 Partial fraction expansions

Rewrite $Y = \frac{(s+2)\,(s+4)}{s\,(s+1)\,(s+3)}$ as a partial fraction expansion and obtain the inverse Laplace transform

Use the command *convert* with the option *parfrac*. There is no menu access to this command.

$Y := \frac{(s+2)\,(s+4)}{s\,(s+1)\,(s+3)}$

$$\frac{(s+2)\,(s+4)}{s\,(s+1)\,(s+3)} \quad \textbf{(3.3.1)}$$

$convert(Y, parfrac)$

$$\frac{8}{3\,s} - \frac{1}{6\,(s+3)} - \frac{3}{2\,(s+1)} \quad \textbf{(3.3.2)}$$

This example uses the equation number **(3.3.2)** i.e. the result of the expansion. Use [Ctl][l] to enter the number. Note that your equation number may vary.

$inttrans[invlaplace](\textbf{(3.3.2)}, s, t)$

$$\frac{8}{3} - \frac{e^{-3t}}{6} - \frac{3\,e^{-t}}{2} \qquad \textbf{(3.3.3)}$$

One possible clickable approach would be to right-click on the expression and then choose *Apply Command*. You would then enter the name of the command (i.e. convert) and the argument (i.e. parfrac). In some ways, this is more cumbersome than simply typing the command.

[Go back to Table of Contents](#)

3.4 Final value theorem

Apply the final value theorem to obtain the steady state response of the system represented by the transfer function $Y(s) = \frac{3(s+2)}{s(s^2+2s+10)}$.

Operation	**Clickable** 
From the *Expression* palette choose [$\lim_{x \to a} f$] and make appropriate substitutions. Note the multiplication by s as required by the theorem. To get a decimal representation of $\frac{3}{5}$ use the right-click menu and choose *Approximate*→5.	$\lim_{s \to 0} \left(\frac{3(s+2)}{s(s^2+2s+10)} \right) s$ $\frac{3}{5}$ **(3.4.1)** $\xrightarrow{\text{at 5 digits}}$ 0.60000 **(3.4.2)**

Command version 

$$Ys := \frac{3(s+2)}{s(s^2+2s+10)}$$

$$\frac{3(s+2)}{s(s^2+2s+10)} \qquad \textbf{(3.4.1.1)}$$

$$yss := limit(s\,Ys, s = 0)$$

$$\frac{3}{5} \qquad \textbf{(3.4.1.2)}$$

$$evalf(yss, 5)$$

$$0.60000 \qquad \textbf{(3.4.1.3)}$$

[Go back to Table of Contents](#)

4. System representation

4.1 Getting the transfer function from the differential equation


Given the differential equation $\ddot{y}(t) + 6\,\dot{y}(t) + 25\,y(t) = 9\,u(t) + 3\,\dot{u}(t)$, determine the transfer function using first principles.

NOTE: you should be able to do this problem by inspection (simply match the coefficients of the derivative terms with appropriate terms in the transfer function). So in reality, you shouldn't be using this technique. However, this example example does illustrate a series of very important Maple techniques that will be useful for other problems. Note that this example is primarily a command-based approach. Although it is theoretically possible to do most with clickable techniques, you will find that they are fairly cumbersome and ultimately will require you to know the command set anyways. In this case the command approach is more intuitive and efficient.

Load the integral transforms library. The list returned is the set of functions now available to you.

$with(inttrans)$

$$[addtable, fourier, fouriercos, fouriersin, hankel, hilbert, invfourier, invhilbert, invlaplace, invmellin, laplace, mellin, savetable] \quad \textbf{(4.1.1)}$$

Define the differential equation. Note that the variables $y(t)$ and $u(t)$ must explicitly be stated as functions of t. No space between y and (t).

$DE := diff(y(t), t, t) + 6\,diff(y(t), t) + 25\,y(t) = 9\,u(t) + 3\,diff(u(t), t)$

$$\frac{d^2}{dt^2}\,y(t) + 6\left(\frac{d}{dt}\,y(t)\right) + 25\,y(t) = 9\,u(t) + 3\left(\frac{d}{dt}\,u(t)\right) \quad \textbf{(4.1.2)}$$

Apply a Laplace transform to the whole equation. Note that the answer is in terms of s and the placeholders in the form $laplace(y(t), t, s)$ which is equivalent to $\mathcal{L}\{y(t)\} = Y(s)$, and initial conditions. Note $D(y)(0)$ is Maple's notation for the first derivative of $y(t)$.

$DEl := laplace(DE, t, s)$

$$s^2\,laplace(y(t), t, s) - D(y)(0) - s\,y(0) + 6\,s\,laplace(y(t), t, s) - 6\,y(0) + 25\,laplace(y(t), t, s) = 9\,laplace(u(t), t, s) + 3\,s\,laplace(u(t), t, s) - 3\,u(0) \quad \textbf{(4.1.3)}$$

Here is a trick to make the equation look more familiar. Replace the place holders for $Y(s)$ and $U(s)$, and apply the usual zero initial conditions.

$DEl := eval(DEl, [laplace(y(t), t, s) = Y(s), laplace(u(t), t, s) = U(s), y(0) = 0, D(y)(0) = 0, u(0) = 0])$

$$s^2\,Y(s) + 6\,s\,Y(s) + 25\,Y(s) = 9\,U(s) + 3\,s\,U(s) \quad \textbf{(4.1.4)}$$

To obtain the transfer function, you need to determine $\frac{Y(s)}{U(s)}$. Do this in two steps: isolate $Y(s)$ then divide the result by $U(s)$. Now simplify to cancel out the several $U(s)$ terms in the expression.

$Ys := isolate(DEl, Y(s))$

$$Y(s) = \frac{9\,U(s) + 3\,s\,U(s)}{s^2 + 6\,s + 25} \quad \textbf{(4.1.5)}$$

$YsUs := \frac{Ys}{U(s)}$

$$\frac{Y(s)}{U(s)} = \frac{9\,U(s) + 3\,s\,U(s)}{U(s)\,(s^2 + 6\,s + 25)} \tag{4.1.6}$$

$TF := simplify(YsUs)$

$$\frac{Y(s)}{U(s)} = \frac{3\,(3 + s)}{s^2 + 6\,s + 25} \tag{4.1.7}$$

The above is in a mathematical equation form (i.e. $a = b$). To get just the expression, extract the right hand side. Here we also overwrite the previous definition of the transfer function.

$TF := rhs(TF)$

$$\frac{3\,(3 + s)}{s^2 + 6\,s + 25} \tag{4.1.8}$$

Go back to Table of Contents

4.2 Using the *DynamicSystems* package for system representations

 Given a differential equation of a system $\ddot{y}(t) + 6\,\dot{y}(t) + 25\,y(t) = 9\,u(t) + 3\,\dot{u}(t)$, use the *DynamicSystems* package to obtain the transfer function and the state space representation.

Load the *DynamicSystems* package

with(*DynamicSystems*)

[*AlgEquation, BodePlot, CharacteristicPolynomial, Chirp, Coefficients, ControllabilityMatrix, Controllable, DiffEquation, DiscretePlot, FrequencyResponse, GainMargin, Grammians, ImpulseResponse, ImpulseResponsePlot, IsSystem, MagnitudePlot, NewSystem, ObservabilityMatrix, Observable, PhaseMargin, PhasePlot, PrintSystem, Ramp, ResponsePlot, RootContourPlot, RootLocusPlot, RouthTable, SSModelReduction, SSTransformation, Simulate, Sinc, Sine, Square, StateSpace, Step, System, SystemOptions, ToDiscrete, TransferFunction, Triangle, Verify, ZeroPoleGain, ZeroPolePlot*] **(4.2.1)**

Assign the differential equation

$DE := diff(y(t), t, t) + 6\,diff(y(t), t) + 25\,y(t) = 9\,u(t) + 3\,diff(u(t), t)$

$$\frac{d^2}{dt^2}\,y(t) + 6\left(\frac{d}{dt}\,y(t)\right) + 25\,y(t) = 9\,u(t) + 3\left(\frac{d}{dt}\,u(t)\right) \tag{4.2.2}$$

Convert to a proper *DynamicSystems* object. Maple returns a summary of the system. You can see the contents with the *PrintSystem* function.

$myDE := DiffEquation(DE, u(t), y(t))$

Diff. Equation
continuous
1 output(s); 1 input(s)
inputvariable = [$u(t)$]
outputvariable = [$y(t)$] **(4.2.3)**

$PrintSystem(myDE)$

$$\left[\begin{array}{c}\textbf{Diff. Equation}\\ \text{continuous}\\ \text{1 output(s); 1 input(s)}\\ \text{inputvariable} = [u(t)]\\ \text{outputvariable} = [y(t)]\\ \text{de} = [\ddot{y}(t) + 6\dot{y}(t) + 25y(t) = 9u(t) + 3\dot{u}(t)]\end{array}\right. \quad (4.2.4)$$

Once you have the system in the right form, you can immediately convert to any of the primary modern forms with a single command. Convert to a transfer function.

$myTF := TransferFunction(myDE)$

$$\left[\begin{array}{c}\textbf{Transfer Function}\\ \text{continuous}\\ \text{1 output(s); 1 input(s)}\\ \text{inputvariable} = [u(s)]\\ \text{outputvariable} = [y(s)]\end{array}\right. \quad (4.2.5)$$

$PrintSystem(myTF)$

$$\left[\begin{array}{c}\textbf{Transfer Function}\\ \text{continuous}\\ \text{1 output(s); 1 input(s)}\\ \text{inputvariable} = [u(s)]\\ \text{outputvariable} = [y(s)]\\ \text{tf}_{1,1} = \dfrac{3s+9}{s^2+6s+25}\end{array}\right. \quad (4.2.6)$$

Generate the required matrices for the state space representation.

$mySS := StateSpace(myTF)$

$$\left[\begin{array}{c}\textbf{State Space}\\ \text{continuous}\\ \text{1 output(s); 1 input(s); 2 state(s)}\\ \text{inputvariable} = [u(t)]\\ \text{outputvariable} = [y(t)]\\ \text{statevariable} = [x1(t), x2(t)]\end{array}\right. \quad (4.2.7)$$

$PrintSystem(mySS)$

State Space

continuous

1 output(s); 1 input(s); 2 state(s)

inputvariable = $[u(t)]$

outputvariable = $[y(t)]$

statevariable = $[x1(t), x2(t)]$

$$a = \begin{bmatrix} 0 & 1 \\ -25 & -6 \end{bmatrix}$$

$$b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$c = \begin{bmatrix} 9 & 3 \end{bmatrix}$$

$$d = \begin{bmatrix} 0 \end{bmatrix} \quad \textbf{(4.2.8)}$$

To get the transfer function expression out of the object, index on the parameter *tf* (i.e.*myTF*[*tf*]). Maple returns, in this case, a 1 × 1 matrix. This is because in complex systems, you can get multiple transfer functions expressed in a matrix. To get the function out as a simple expression, extract the [1,1] element of this matrix.

myTF[*tf*]

$$\begin{bmatrix} \frac{3s+9}{s^2+6s+25} \end{bmatrix} \quad \textbf{(4.2.9)}$$

myTF[*tf*] [1, 1]

$$\frac{3s+9}{s^2+6s+25} \quad \textbf{(4.2.10)}$$

To access the statespace matrices, A, B, C, and D, index on the corresponding name in lower case. This example shows how to get the matrix and use it to compose a statespace matrix expression.   Note, Maple chooses a different term order in the final equation. There's really not much you can do about that without resorting to very low-level programming.

mySS[*a*]

$$\begin{bmatrix} 0 & 1 \\ -25 & -6 \end{bmatrix} \quad \textbf{(4.2.11)}$$

mySS[*b*]

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \textbf{(4.2.12)}$$

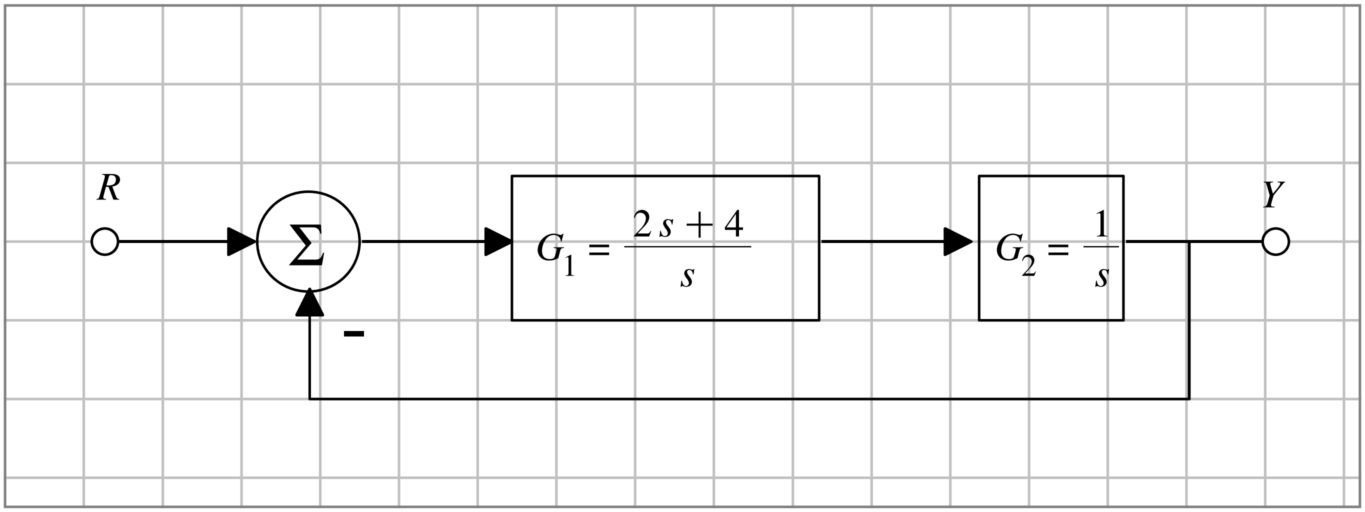
$$\begin{bmatrix} \frac{d}{dt} x_1(t) \\ \frac{d}{dt} x_2(t) \end{bmatrix} = mySS[a] . \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + mySS[b] . u(t)$$

$$\begin{bmatrix} \frac{d}{dt} x_1(t) \\ \frac{d}{dt} x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} . u(t) + \begin{bmatrix} x_2(t) \\ -25\, x_1(t) - 6\, x_2(t) \end{bmatrix} \quad \textbf{(4.2.13)}$$

Go back to Table of Contents

4.3 Block diagrams

Closed loop transfer function for the following feedback system is $\dfrac{G_1 G_2}{1 + G_1 G_2}$.



Operation	**Clickable** 
Define block expressions. To use equation numbers, type [Ctl][l] (lower case L), then enter the equation number (note that yours may be different than this example). For simplification, use Right Click menu.	$\dfrac{2\,s+4}{s}$ $\dfrac{2\,s+4}{s}$ **(4.3.1)** $\dfrac{1}{s}$ $\dfrac{1}{s}$ **(4.3.2)** $\dfrac{\textbf{(4.3.1)(4.3.2)}}{1 + \textbf{(4.3.1)(4.3.2)}}$ $\dfrac{2\,s+4}{s^2\left(1+\dfrac{2\,s+4}{s^2}\right)}$ **(4.3.3)** $\overset{\text{simplify}}{=}$ $\dfrac{2\,(s+2)}{s^2+2\,s+4}$

Command version 

$$G1 := \frac{2\,s+4}{s}$$

$$G1 := \frac{2\,s+4}{s} \qquad \textbf{(4.3.1.1)}$$

$G2 := \frac{1}{s}$

$$G2 := \frac{1}{s} \tag{4.3.1.2}$$

$TF := \frac{G1\,G2}{1 + G1\,G2}$

$$TF := \frac{2\,s + 4}{s^2 \left(1 + \frac{2\,s + 4}{s^2}\right)} \tag{4.3.1.3}$$

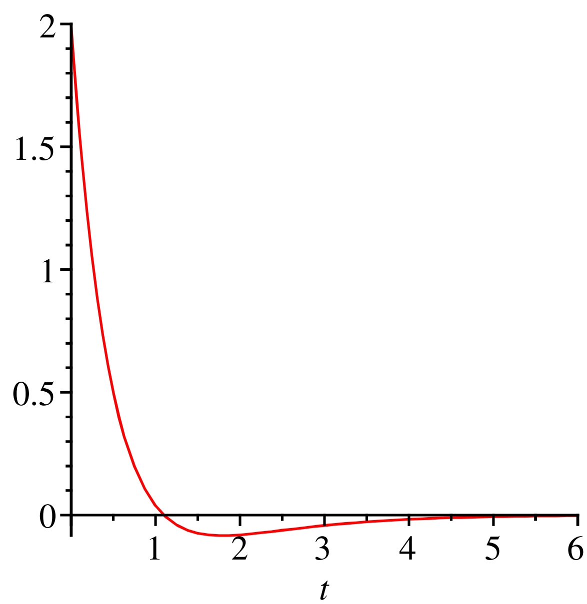
$TFs := simplify(TF)$

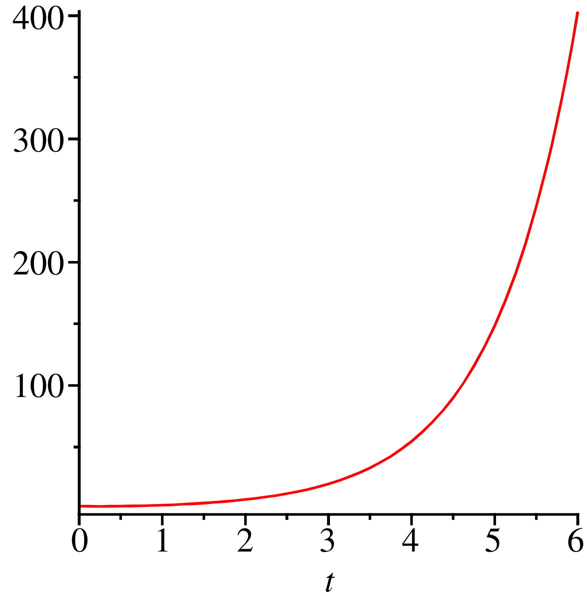
$$TFs := \frac{2\,(s + 2)}{s^2 + 2\,s + 4} \tag{4.3.1.4}$$

Go back to Table of Contents

5. System response

5.1 Pole location and transient response

Operation	Clickable
Define transfer function then use right click menu *Factor*. Compute inverse Laplace transform by menu *Integral Transforms→Laplace*.Plot with menu *Plot Builder*. Ensure that plot time is 0 to 6.	$\frac{2s+1}{s^2+3s+2}$ $\frac{2s+1}{s^2+3s+2}$ **(5.1.1)** $\overset{\text{factor}}{=}$ $\frac{2s+1}{(s+2)(s+1)}$ **(5.1.2)** $\xrightarrow{\text{inverse Laplace transform}}$ $3\,e^{-2t} - e^{-t}$ **(5.1.3)** $\rightarrow$ 
Copy and paste factored transfer function above to a new math line. Change second factor to $(s-1)$. Repeat steps and plot. To explore different real values of poles, simply change the factors and repeat the steps.	$\frac{2s+1}{(s+2)(s-1)}$ $\frac{2s+1}{(s+2)(s-1)}$ **(5.1.4)** $\xrightarrow{\text{inverse Laplace transform}}$ $e^{t} + e^{-2t}$ **(5.1.5)** $\rightarrow$



To try complex poles, change the factor value to a complex number. Make sure that the two are complex conjugate pairs -- i.e. they only differ by the sign of the imaginary term.

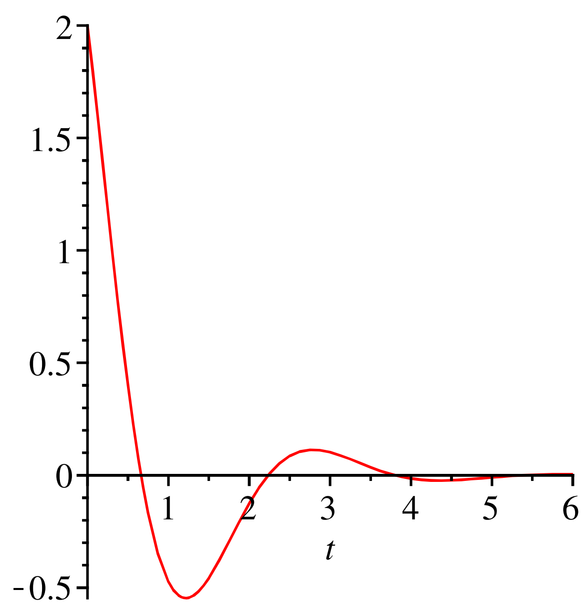
$$\frac{2\,s+1}{(s+1+2\,I)\,(s+1-2\,I)}$$

$$\frac{2\,s+1}{(s+1+2\,\mathrm{I})\,(s+1-2\,\mathrm{I})} \tag{5.1.6}$$

$\xrightarrow{\text{inverse Laplace transform}}$

$$\frac{\mathrm{e}^{-t}\,(4\cos(2\,t)-\sin(2\,t))}{2} \tag{5.1.7}$$

$\rightarrow$



Command version

$$H := \frac{2\,s+1}{s^2+3\,s+2}$$

$$\frac{2s+1}{s^2+3s+2} \quad \textbf{(5.1.1.1)}$$

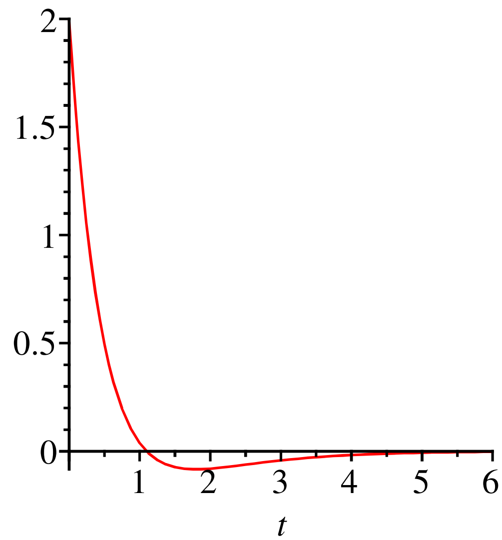
$Hf := factor(H)$

$$\frac{2s+1}{(s+2)(s+1)} \quad \textbf{(5.1.1.2)}$$

$ht := inttrans[invlaplace](Hf, s, t)$

$$3\,e^{-2t} - e^{-t} \quad \textbf{(5.1.1.3)}$$

$plot(ht, t = 0..6)$

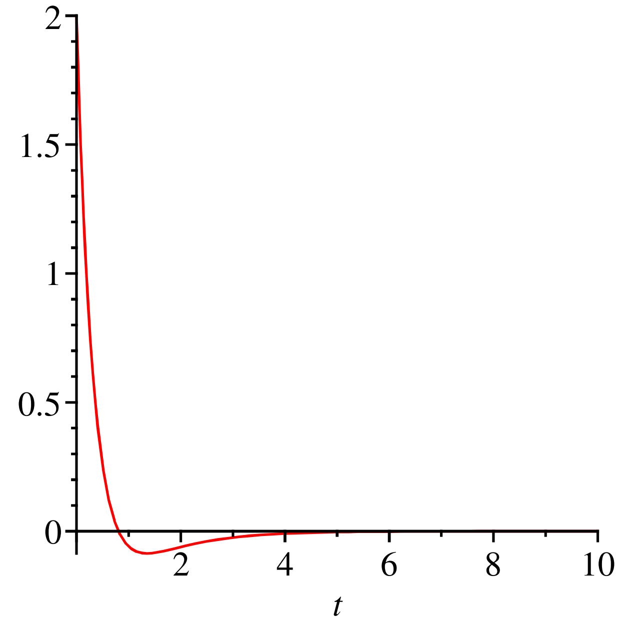
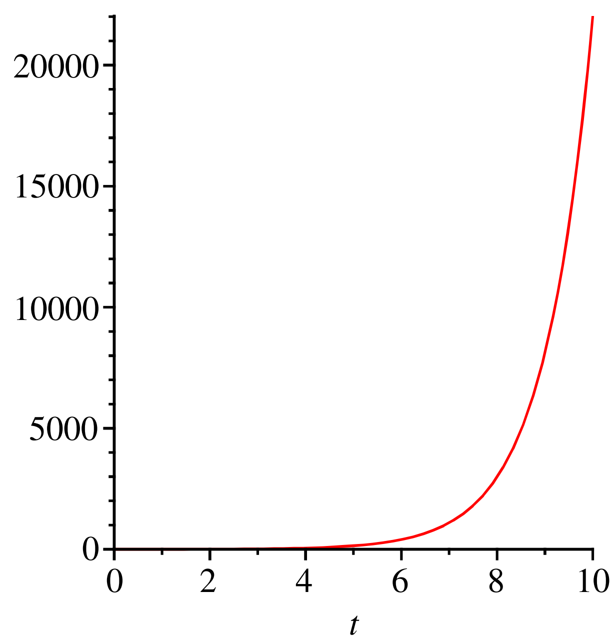


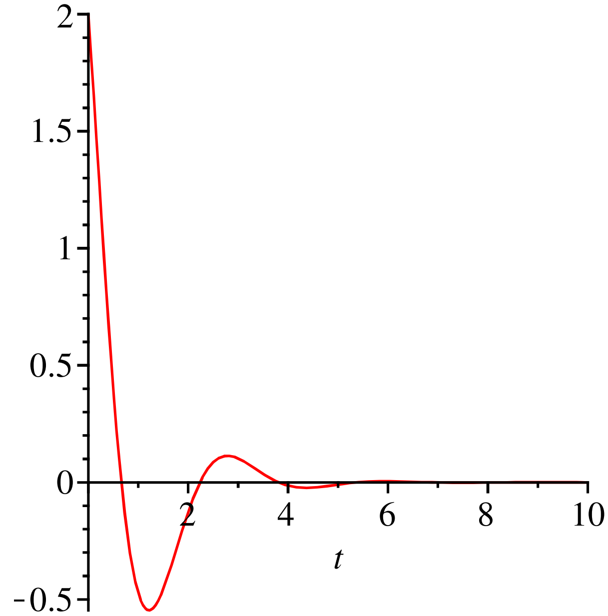
Go back to Table of Contents

5.1.1 Building exploration tools

By using operators (functions), and other convenient Maple tools, you can easily try out different values of system parameters. This first example converts the previous example to operator form.

Operation	Clickable
Using the *Expression Palette*, choose the 2 parameter operator tool [$f := (a, b) \rightarrow z$] and change the name from *f* to *Hf* and the expression *z*.	$Hf := (a, b) \rightarrow \frac{2s+1}{(s+a)(s+b)}$ $(a, b) \rightarrow \frac{2s+1}{(s+a)(s+b)}$ **(5.1.2.1)**
Now you can use the function $Hf(a, b)$ for different values of the poles. For example for the values 3 and 1. Use *Inverse Laplace* from menu and then *Plot Builder* with range 0 to 10.	$Hf(3, 1)$ $\frac{2s+1}{(s+3)(s+1)}$ **(5.1.2.2)** $\xrightarrow{\text{inverse Laplace transform}}$ $e^{-2t}(2\cosh(t) - 3\sinh(t))$ **(5.1.2.3)** $\rightarrow$

	
It will now work for any values. Note what happens when $b < 0$. This implies the pole is on the right half plane and therefore the system is unstable.	$Hf(2,-1)$ $$\frac{2s+1}{(s+2)(s-1)} \quad \textbf{(5.1.2.4)}$$ $\xrightarrow{\text{inverse Laplace transform}}$ $$e^{t} + e^{-2t} \quad \textbf{(5.1.2.5)}$$ $\rightarrow$ 

Maple is smart enough to deal with complex values of *a* and *b*. Make sure they are complex conjugates -- i. e. differ only be the sign of the imaginary term. The complex poles will result in sines and cosines in the time response which will result in increased oscillation.	$Hf(1+2I, 1-2I)$ $\frac{2s+1}{(s+1+2\mathrm{I})(s+1-2\mathrm{I})}$ **(5.1.2.6)** $\xrightarrow{\text{inverse Laplace transform}}$ $\frac{e^{-t}(4\cos(2t)-\sin(2t))}{2}$ **(5.1.2.7)** $\rightarrow$ 

Command version 

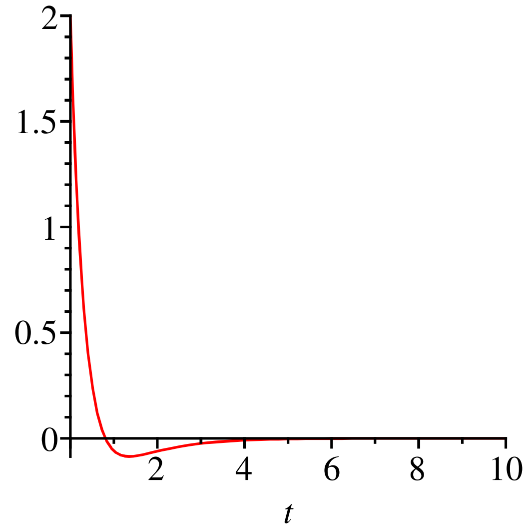
$case1 := Hf(3, 1)$

$$\frac{2s+1}{(s+3)(s+1)} \quad \textbf{(5.1.2.1.1)}$$

$yt := inttrans[invlaplace](case1, s, t)$

$$e^{-2t}(2\cosh(t) - 3\sinh(t)) \quad \textbf{(5.1.2.1.2)}$$

$plot(yt, t=0..10)$



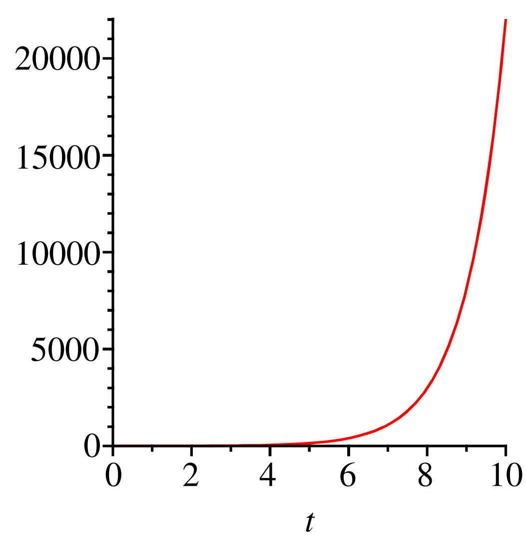
case2 := *Hf*(2, −1)

$$\frac{2\,s+1}{(s+2)\,(s-1)} \qquad \textbf{(5.1.2.1.3)}$$

yt := *inttrans*[*invlaplace*](*case2*, *s*, *t*)

$$e^{-2\,t}+e^{t} \qquad \textbf{(5.1.2.1.4)}$$

plot(*yt*, *t* = 0 ..10)



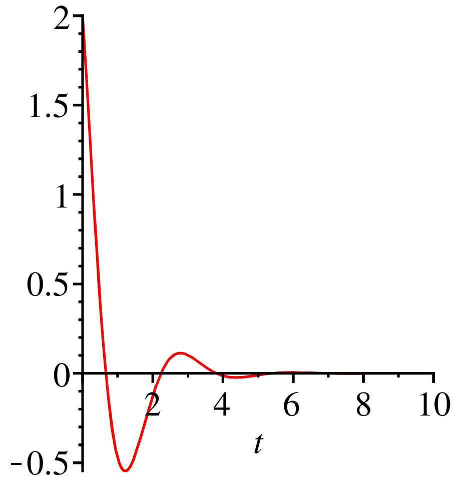
case3 := *Hf*(1 + 2 *I*, 1 − 2 *I*)

$$\frac{2\,s+1}{(s+1+2\,\mathrm{I})\,(s+1-2\,\mathrm{I})} \qquad \textbf{(5.1.2.1.5)}$$

yt := *inttrans*[*invlaplace*](*case3*, *s*, *t*)

$$\frac{1}{2}\,e^{-t}\,(4\cos(2\,t)-\sin(2\,t)) \qquad \textbf{(5.1.2.1.6)}$$

plot(*yt*, *t* = 0 ..10)



Version 2: More elaborate function 

Combine the full inverse Laplace transform command inside your operator function.

$Hf2 := (a, b) \rightarrow inttrans[invlaplace]\left(\frac{2\,s+1}{(s+a)\,(s+b)}, s, t\right)$

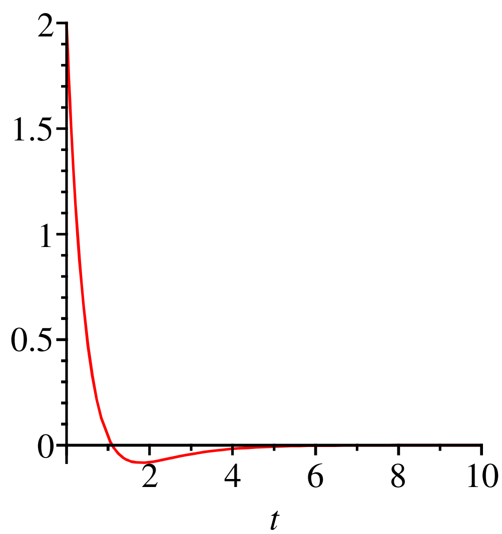
$$(a, b) \rightarrow inttrans_{invlaplace}\left(\frac{2\,s+1}{(s+a)\,(s+b)}, s, t\right) \tag{5.1.2.2.1}$$

Now when you call the function, you get the response expression in one step. You can now plot. Note, you could have placed the plot command inside the operator giving you a one step option right to the plot.

$yt := Hf2(2, 1)$

$$-e^{-t} + 3\,e^{-2\,t} \tag{5.1.2.2.2}$$

$plot(yt, t = 0\,..10)$



Go back to Table of Contents

5.2 Stability: Routh-Hurwitz criterion with the *DynamicSystems* package

For the characteristic equation, $s^6 + 4\,s^5 + 3\,s^4 + 2\,s^3 + s^2 + 4\,s + 4$, generate the Routh table.

Load *DynamicSystems* package.

$with(DynamicSystems)$

[*AlgEquation, BodePlot, CharacteristicPolynomial, Chirp, Coefficients, ControllabilityMatrix, Controllable, DiffEquation, DiscretePlot, FrequencyResponse, GainMargin, Grammians, ImpulseResponse, ImpulseResponsePlot, IsSystem, MagnitudePlot, NewSystem, ObservabilityMatrix, Observable, PhaseMargin, PhasePlot, PrintSystem, Ramp, ResponsePlot, RootContourPlot, RootLocusPlot, RouthTable, SSModelReduction, SSTransformation, Simulate, Sinc, Sine, Square, StateSpace, Step, System, SystemOptions, ToDiscrete, TransferFunction, Triangle, Verify, ZeroPoleGain, ZeroPolePlot*] **(5.2.1)**

Define characteristic equation.

$mypoly := s^6 + 4\,s^5 + 3\,s^4 + 2\,s^3 + s^2 + 4\,s + 4$

$$s^6 + 4\,s^5 + 3\,s^4 + 2\,s^3 + s^2 + 4\,s + 4 \qquad \textbf{(5.2.2)}$$

Use *RouthTable* command.

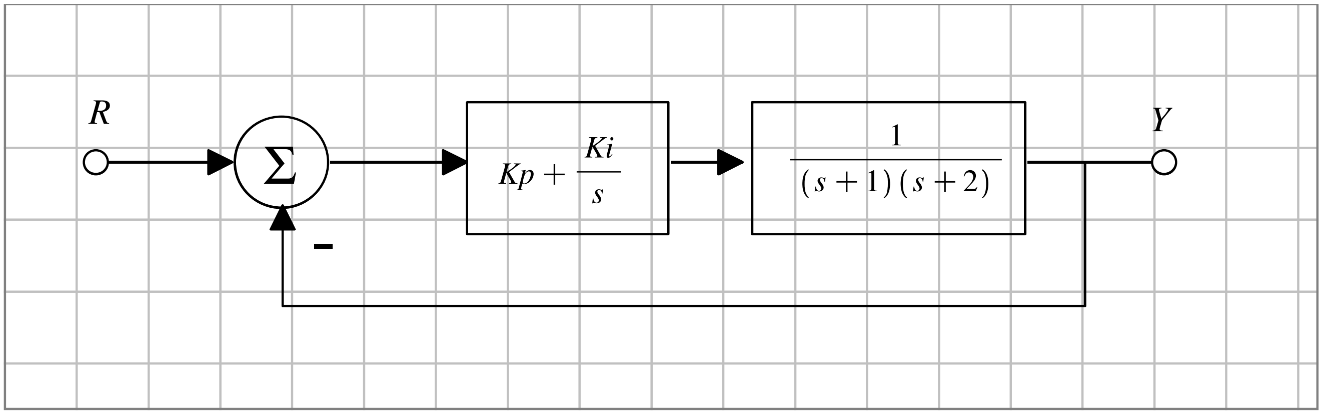
$RouthTable(mypoly, s)$

$$\begin{bmatrix} 1 & 3 & 1 & 4 & s^6 \\ 4 & 2 & 4 & 0 & s^5 \\ \frac{5}{2} & 0 & 4 & 0 & s^4 \\ 2 & -\frac{12}{5} & 0 & 0 & s^3 \\ 3 & 4 & 0 & 0 & s^2 \\ -\frac{76}{15} & 0 & 0 & 0 & s \\ 4 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \textbf{(5.2.3)}$$

Go back to Table of Contents

5.2.1 Example: determining conditions for stability (Routh criterion)

For the following feedback control system (PI controller), determine the conditions on *Kp* and *Ki* for which the system is stable.



Define transfer function as an equation $TF(s) = 0$. Maple does some initial rearrangement.

$TF := 1 + \left(Kp + \frac{Ki}{s}\right)\left(\frac{1}{(s+1)(s+2)}\right) = 0$

$$1 + \frac{Kp + \frac{Ki}{s}}{(s+1)(s+2)} = 0 \quad \textbf{(5.2.1.1)}$$

simplify to get into a single rational on the left hand side.

$TF := simplify(TF)$

$$\frac{s^3 + 3s^2 + 2s + Kp\,s + Ki}{s(s+1)(s+2)} = 0 \quad \textbf{(5.2.1.2)}$$

Extract the numerator of the left hand side of the above equation. This is the characteristic equation.

$chareq := numer(lhs(TF))$

$$s^3 + 3s^2 + 2s + Kp\,s + Ki \quad \textbf{(5.2.1.3)}$$

Use *RouthTable* command as before leaving parameters undefined. The result is not completely useful as you will need to work with the elements to sort out conditions on *Kp* and *Ki*.

with(DynamicSystems)

[*AlgEquation*, *BodePlot*, *CharacteristicPolynomial*, *Chirp*, *Coefficients*, *ControllabilityMatrix*, *Controllable*, *DiffEquation*, *DiscretePlot*, *FrequencyResponse*, *GainMargin*, *Grammians*, *ImpulseResponse*, *ImpulseResponsePlot*, *IsSystem*, *MagnitudePlot*, *NewSystem*, *ObservabilityMatrix*, *Observable*, *PhaseMargin*, *PhasePlot*, *PrintSystem*, *Ramp*, *ResponsePlot*, *RootContourPlot*, *RootLocusPlot*, *RouthTable*, *SSModelReduction*, *SSTransformation*, *Simulate*, *Sinc*, *Sine*, *Square*, *StateSpace*, *Step*, *System*, *SystemOptions*, *ToDiscrete*, *TransferFunction*, *Triangle*, *Verify*, *ZeroPoleGain*, *ZeroPolePlot*] **(5.2.1.4)**

RouthTable(chareq, s)

$$\begin{bmatrix} 1 & 2 + Kp & s^3 \\ 3 & Ki & s^2 \\ 2 + Kp - \frac{1}{3}Ki & 0 & s \\ Ki & 0 & 1 \end{bmatrix}$$

RouthTable(chareq, s, stablecondition = true)

$$0 < Ki \ \textbf{and} \ 0 < 6 + 3\,Kp - Ki \quad \textbf{(5.2.1.6)}$$

If you set the *stablecondition* option, you will get the conditions on your parameters that you will need to resolve. They appear as inequalities.

ineq := $0 < 6 + 3\,Kp - Ki$

$$0 < 6 + 3\,Kp - Ki \qquad \textbf{(5.2.1.7)}$$

Maple can also solve inequalities. The result here is the condition relating *Ki* and *Kp* for which the system is stable.

solve(*ineq*, *Kp*)

$$\left\{ -2 + \frac{1}{3}\,Ki < Kp \right\} \qquad \textbf{(5.2.1.8)}$$

Go back to Table of Contents

5.3 Root locus methods

Maple offers some very convenient tools for generating root locus plots and exploring the many useful concepts embodied in the technique.

5.3.1 Root locus plot with the DynamicSystems package 

Load the *DynamicSystems* package.

with(*DynamicSystems*)

[*AlgEquation*, *BodePlot*, *CharacteristicPolynomial*, *Chirp*, *Coefficients*, *ControllabilityMatrix*, *Controllable*, *DiffEquation*, *DiscretePlot*, *FrequencyResponse*, *GainMargin*, *Grammians*, *ImpulseResponse*, *ImpulseResponsePlot*, *IsSystem*, *MagnitudePlot*, *NewSystem*, *ObservabilityMatrix*, *Observable*, *PhaseMargin*, *PhasePlot*, *PrintSystem*, *Ramp*, *ResponsePlot*, *RootContourPlot*, *RootLocusPlot*, *RouthTable*, *SSModelReduction*, *SSTransformation*, *Simulate*, *Sinc*, *Sine*, *Square*, *StateSpace*, *Step*, *System*, *SystemOptions*, *ToDiscrete*, *TransferFunction*, *Triangle*, *Verify*, *ZeroPoleGain*, *ZeroPolePlot*] **(5.3.1.1)**

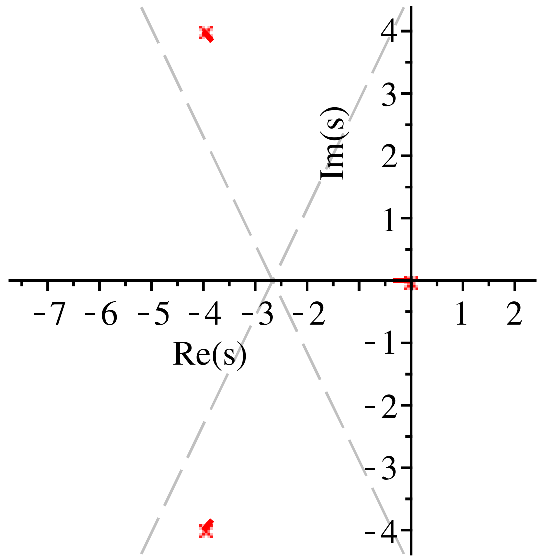
Define a new system using a transfer function. The *NewSystem* command returns a variation of the transfer function that is compatible with the other commands in the *DynamicSystems* package.

$G1 := NewSystem\left(\frac{1}{s\,(s^2 + 8\,s + 32)} \right)$

Transfer Function
continuous
1 output(s); 1 input(s)
inputvariable = [$u1(s)$]
outputvariable = [$y1(s)$] **(5.3.1.2)**

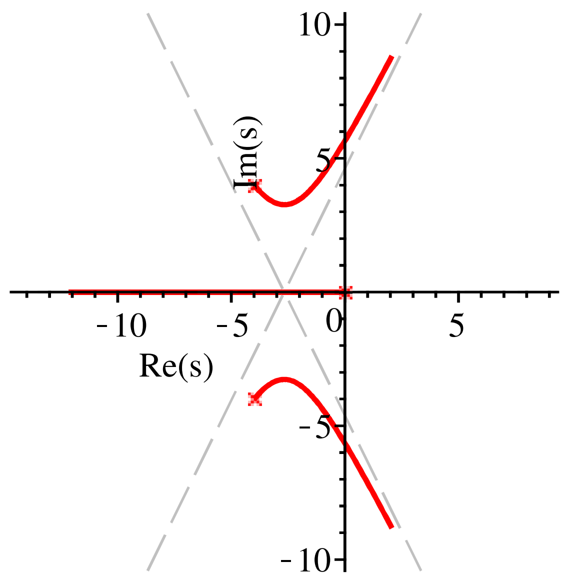
Call *RootLocusPlot* command. Default gain range is 0 to 10. In this case, the gain range is insufficient to provide meaningful root locus information.

RootLocusPlot(*G1*)



Try again for a gain range of 0 to 1000. The resulting plot is more complete.

RootLocusPlot(*G1*, 0 ..1000)

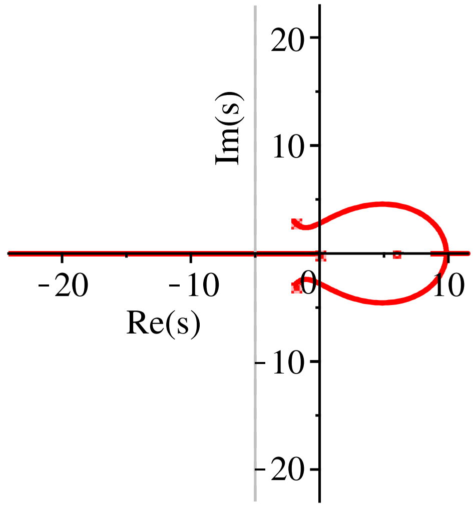


Using the option *info=rlinfo*, you can get useful, specific information. For example, the the *Kbranches* provides the value of the gain where the locus splits.

$G2 := NewSystem\left(\frac{(s-6)}{s\,(s^2+4\,s+13)} \right)$

$$\left[\begin{array}{c} \textbf{Transfer Function} \\ \text{continuous} \\ \text{1 output(s); 1 input(s)} \\ \text{inputvariable} = [\,u1(s)\,] \\ \text{outputvariable} = [\,y1(s)\,] \end{array} \right. \quad \textbf{(5.3.1.3)}$$

RootLocusPlot(*G2*, −400 ..0, *info* = *rlinfo*)



$print(rlinfo)$

$$Record\left(deq = \left(deq = \left(\frac{d}{dK}\, s(K) = -\frac{s(K) - 6.}{s(K)^2 + 4.\, s(K) + 13. + s(K)\,(2\, s(K) + 4.) + K}\right)\right), charpoly = s\,(s^2 + 4.\, s + 13.) + K\,(s - 6.), G = \frac{s - 6.}{s^3 + 4.\, s^2 + 13.\, s}, Kcrit = (\ ), Kbranches = [-382.2880836], zeros = [6.], poles = [-2.000000000 - 3.000000000\,I, -2. + 3.000000000\,I, 0.]\right) \quad \textbf{(5.3.1.4)}$$

$rlinfo[Kbranches]$

$$[-382.2880836] \quad \textbf{(5.3.1.5)}$$

rlinfo:-*Kbranches*

$$[-382.2880836] \quad \textbf{(5.3.1.6)}$$

[Go back to Table of Contents]

5.3.2 Discrete point approach to creating a root locus plot

This example uses basic principles to create a root locus plot. A root locus plot traces the poles of the closed loop transfer function as some parameter (typically forward path gain K) varies within a range. Each time K changes, it changes the characteristic equation which then produces different roots (poles). So by systematically changing values of K and solving the resulting characteristic equation, then plotting the real and imaginary parts on a plot grid, you can construct a root locus plot point by point. Some of the advantages of this construction are greater flexibility, i.e. you can choose different parameters that impact the characteristic equation and not just the forward path gain, and the spacing of the plot points gives you valuable sensitivity information. That is, if a change in K produces pole positions that are far apart, this suggests the system is sensitive in that parameter range.

Define transfer function, and extract the characteristic polynomial. Note that we do not need to load the *DynamicSystems* package as we are "manually" constructing the root locus.

$$G := \frac{K}{s\,(s^2 + 8\,s + 32)}$$

$$\frac{K}{s\,(s^2 + 8\,s + 32)} \quad \textbf{(5.3.2.1)}$$

$$TF := simplify\left(\frac{K\,G}{1 + K\,G}\right)$$

$$\frac{K^2}{s^3 + 8\,s^2 + 32\,s + K^2} \quad \textbf{(5.3.2.2)}$$

$$cpoly := denom(TF)$$

$$s^3 + 8\,s^2 + 32\,s + K^2 \quad \textbf{(5.3.2.3)}$$

String together a "sequence" of the characteristic polynomials. A sequence is a Maple datastructure that is a collection of objects separated by a comma. E.g. a,b,c is a sequence. You can compose a sequence efficiently using the *seq* command.

The result here is a sequence of all variations of the characteristic polynomial as *K* varies from 0 to 10 in increments of 1.

$seq(cpoly, K = 0..10)$

$$s^3 + 8s^2 + 32s,\ s^3 + 8s^2 + 32s + 1,\ s^3 + 8s^2 + 32s + 4,\ s^3 + 8s^2 + 32s + 9,\ s^3 + 8s^2 + 32s + 16,\ s^3 + 8s^2 + 32s + 25,\ s^3 + 8s^2 + 32s + 36,\ s^3 + 8s^2 + 32s + 49,\ s^3 + 8s^2 + 32s + 64,\ s^3 + 8s^2 + 32s + 81,\ s^3 + 8s^2 + 32s + 100 \quad \textbf{(5.3.2.4)}$$

Try building a different sequence. Here, instead of stringing together the characteristic polynomials, try solving the polynomial and string together the computed roots. So each iteration of your sequence would have to do something like the following:

$cpolytmp := eval(cpoly, K = 2)$

$$s^3 + 8s^2 + 32s + 4 \quad \textbf{(5.3.2.5)}$$

$solve(cpolytmp)$

$$\frac{1}{3}\left(586 + 54\sqrt{129}\right)^{1/3} - \frac{32}{3\left(586 + 54\sqrt{129}\right)^{1/3}} - \frac{8}{3},\ -\frac{1}{6}\left(586 + 54\sqrt{129}\right)^{1/3} + \frac{16}{3\left(586 + 54\sqrt{129}\right)^{1/3}} - \frac{8}{3} + \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(586 + 54\sqrt{129}\right)^{1/3} + \frac{32}{3\left(586 + 54\sqrt{129}\right)^{1/3}}\right),\ -\frac{1}{6}\left(586 + 54\sqrt{129}\right)^{1/3} + \frac{16}{3\left(586 + 54\sqrt{129}\right)^{1/3}} - \frac{8}{3} - \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{3}\left(586 + 54\sqrt{129}\right)^{1/3} + \frac{32}{3\left(586 + 54\sqrt{129}\right)^{1/3}}\right) \quad \textbf{(5.3.2.6)}$$

The above commands will show you what one particular step in this sequence will give. The result of the complete sequence command is a very long sequence of complex numbers that represent the roots of all of the characteristic polynomials. For brevity in this print version of this guide, most of the output is omitted.

$seq(solve(cpoly), K = 0..10)$

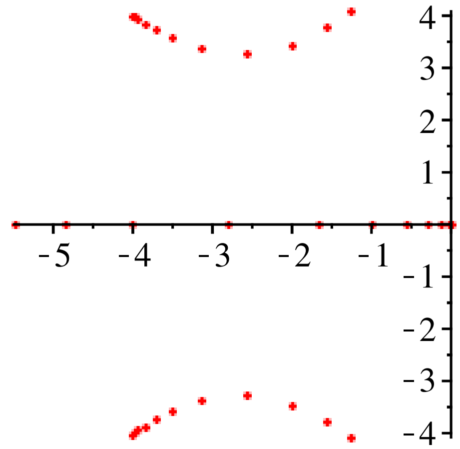
$$0, -4 + 4\,\mathrm{I}, -4 - 4\,\mathrm{I},\ \frac{1}{6}\left(5012 + 36\sqrt{21001}\right)^{1/3} - \frac{64}{3\left(5012 + 36\sqrt{21001}\right)^{1/3}} - \frac{8}{3}, -\frac{1}{12}\left(5012 + 36\sqrt{21001}\right)^{1/3} + \frac{32}{3\left(5012 + 36\sqrt{21001}\right)^{1/3}} - \frac{8}{3} + \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{6}\left(5012 + 36\sqrt{21001}\right)^{1/3} + \frac{64}{3\left(5012 + 36\sqrt{21001}\right)^{1/3}}\right), -\frac{1}{12}\left(5012 + 36\sqrt{21001}\right)^{1/3} + \frac{32}{3\left(5012 + 36\sqrt{21001}\right)^{1/3}} - \frac{8}{3} - \frac{1}{2}\,\mathrm{I}\sqrt{3}\left(\frac{1}{6}\left(5012 + 36\sqrt{21001}\right)^{1/3} + \frac{64}{3\left(5012 + 36\sqrt{21001}\right)^{1/3}}\right) \ldots$$

... rest of the output is deleted for brevity

Using the *complexplot* command contained in the *plots* package, each complex number will plot as a point. For example, a value of $1 + 2I$ will plot as the point (1,2). Make sure you invoke the option *style=point.*

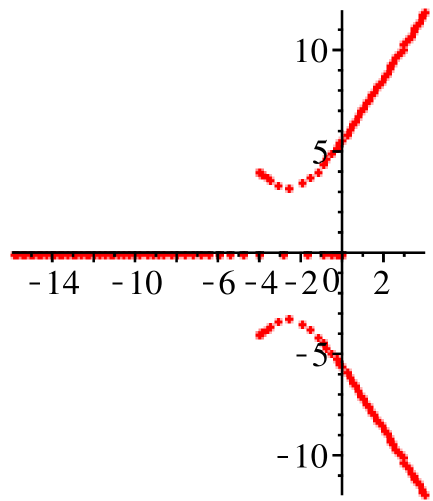
We now see some type of a root locus. The detail is pretty sparse though.

$plots[complexplot]([seq(solve(cpoly, s), K = 0..10)], style = point)$



Try the same thing but use a longer range of *K*. This time to 50.

plots[*complexplot*]([*seq*(*solve*(*cpoly*, *s*) , *K* = 0 ..50)], *style* = *point*)

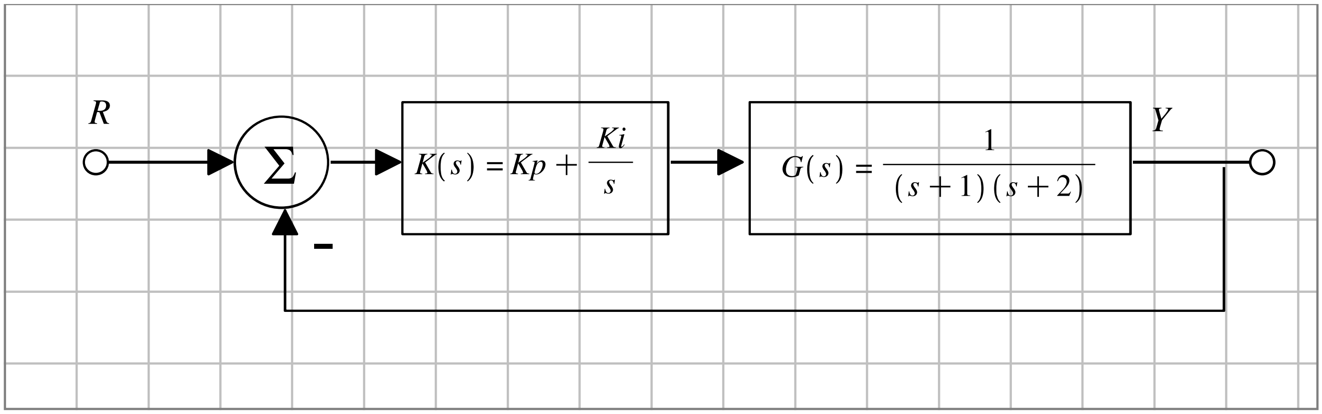


[*Go back to Table of Contents*]

5.3.3 Discrete point example 2: exploring 2 parameters

This is another example that takes the basic definition of the root locus (the trace of the roots of the characteristic equation as a parameter varies) and extends it to an analysis that is typically not easily achieved with more conventional treatments of the root locus.

Consider again the following feedback system with a PI control:



The closed loop transfer function is $\frac{K(s)\,G(s)}{1+K(s)\,G(s)}$ and consequently, the characteristic equation $1+K(s)\,G(s)$ is dependent on both *Kp* and *Ki*. The construction of the root locus from conventional methods will present several algebraic hurdles to sort through the math. The following simply adapts the exact same discrete point technique of the previous section in a way that visually treats the two parameters

Define closed loop transfer function and obtain the characteristic equation which is dependent on *Kp* and *Ki*.

$$TF := simplify\left(\frac{\left(Kp+\frac{Ki}{s}\right)\left(\frac{1}{(s+1)\,(s+2)}\right)}{1+\left(Kp+\frac{Ki}{s}\right)\left(\frac{1}{(s+1)\,(s+2)}\right)}\right)$$

$$\frac{Kp\,s+Ki}{s^3+3\,s^2+2\,s+Kp\,s+Ki} \qquad \textbf{(5.3.3.1)}$$

$cpoly := denom(TF)$

$$s^3+3\,s^2+2\,s+Kp\,s+Ki \qquad \textbf{(5.3.3.2)}$$

Now define a Maple function that takes as input, *Ki* (using temporary variable name *ki* - lower case *k*), substitutes into the characteristic equation (which now becomes only a function of *Kp* since *Ki* has a value), then constructs a sequence within which the value of *Kp* is varied from 0 to 50 and for each value, you solve for the roots and through the sequence, you string together all of the solved roots. After this, you then pour the roots into the *complexplot* routine which will then create and return a plot data structure.

Remember this is a function definition so you will not get any results, just an echo of the function.

$RLF := (ki) \rightarrow plots[complexplot]([seq(solve(eval(cpoly, Ki = ki), s), Kp = 0 ..50)], style = point)$

$$ki \rightarrow plots_{complexplot}\left(\left[seq\left(solve\left(cpoly\Big|_{Ki = ki}, s\right), Kp = 0 ..50\right)\right], style = point\right) \qquad \textbf{(5.3.3.3)}$$

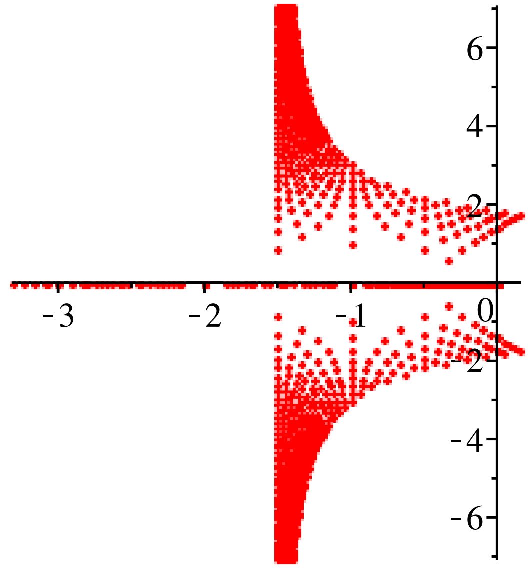
Now that you have a function that returns a plot for each value of *Ki*, let's use this to string together a whole series of plots for different values of *ki*. To do this, you build another sequence, but this time you string entire plots together. At the end, you will have a total of 11 plots in datastructure form.

$plotseq := seq(RLF(ki), ki = 0 ..10)$

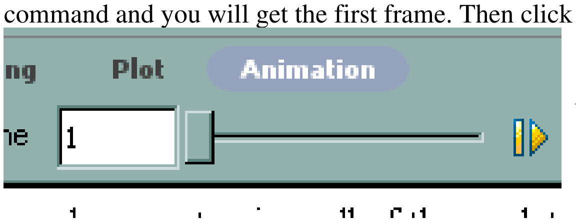
$PLOT(...), PLOT(...), PLOT(...), PLOT(...), PLOT(...), PLOT(...), PLOT(...), PLOT(...), PLOT(...), PLOT(...), PLOT(...)$ **(5.3.3.4)**

Now you need a way to view all of these plots. By using the *display* function in the *plots* package. The default simply shows all of the plots together at the same time.

$plots[display](plotseq)$

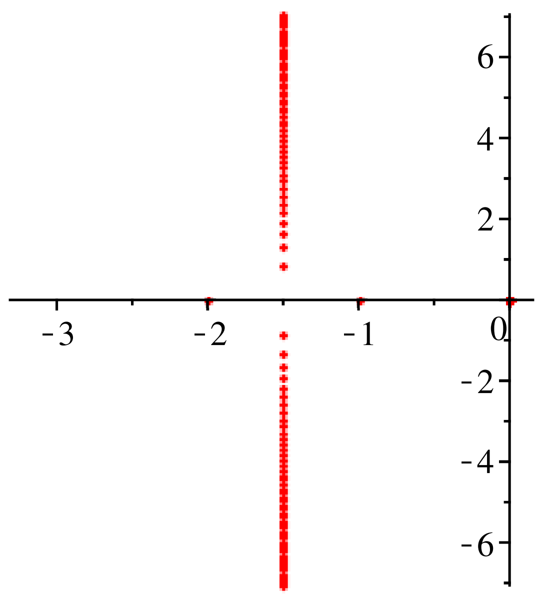


With an *insequence=true* setting, you can create an animation that treats each plot as a frame. Execute this command and you will get the first frame. Then click on the plot. You will then see a slider control on the tool bar.



Use this control to move the animation.

plots[*display*](*plotseq*, *insequence* = *true*)



If you survived this exercise and you are comfortable with all of the commands and the steps, you are well on your way to becoming a Maple master! Being able to freely and efficiently mix math, graphics, programming, and interactive tools to solve complex problems that are typically impossible with traditional math or programming systems is really the most powerful Maple skill that you can achieve. Congratulations ... it is now time for you to leave Grasshopper ... for and explanation of this obscure reference, see http://www.imdb.com/title/tt0068823/quotes and http://en.wikipedia.

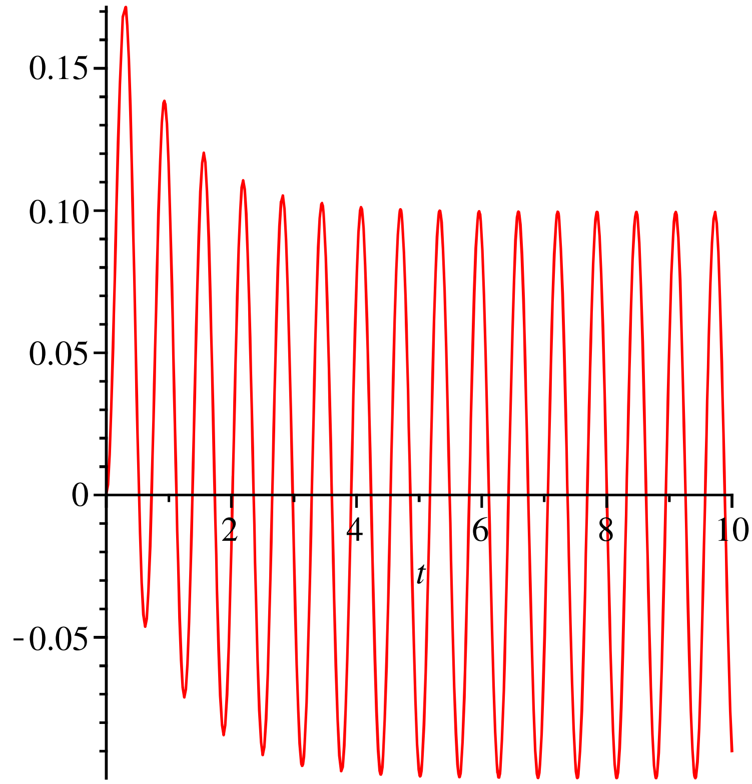
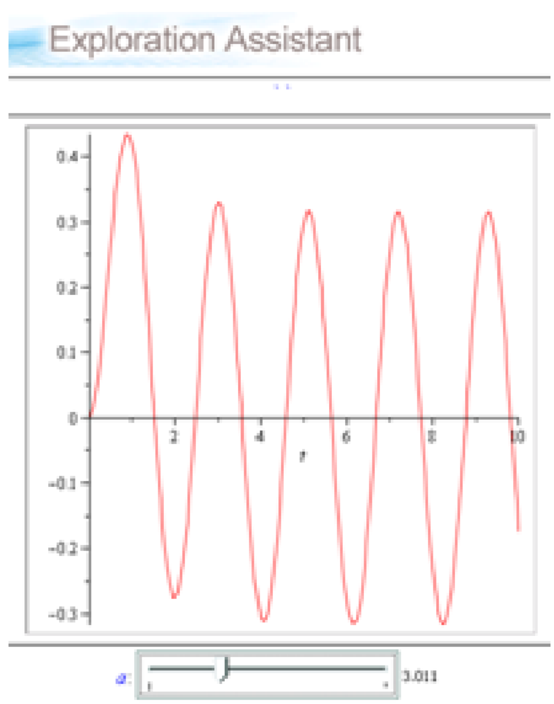
org/wiki/Kung_Fu_(TV_series) ☺

Go back to Table of Contents

5.4 Frequency domain analysis: Bode plots

5.4.1 Example: Time domain response to input $\sin(a\,t)$

Operation	Clickable
Define transfer function $G(s)$.	$G := \frac{1}{s+1}$ $\frac{1}{s+1}$ **(5.4.1.1)**
Define the input function as a sine wave with a frequency parameter a. Get the inverse Laplace transform through the menu and assign to a name Rs.	$rt := \sin(a\,t)$ $\sin(a\,t)$ **(5.4.1.2)** $\xrightarrow{\text{Laplace transform}}$ $\frac{a}{s^2+a^2}$ **(5.4.1.3)** $\xrightarrow{\text{assign to a name}}$ $\frac{a}{s^2+a^2}$ **(5.4.1.4)**
The output in Laplace domain will be the product of the system transfer function and the input. Compute the product, get the inverse Laplace transform. This is now the general formula of the response of the system to a general sine wave input. Give this a name *yt*.	$Y := G\,Rs$ $\frac{a}{(s+1)\,(s^2+a^2)}$ **(5.4.1.5)** $\xrightarrow{\text{inverse Laplace transform}}$ $\frac{\sin(a\,t)+(\mathrm{e}^{-t}-\cos(a\,t))\,a}{1+a^2}$ **(5.4.1.6)** $\xrightarrow{\text{assign to a name}}$ $\frac{\sin(a\,t)+(\mathrm{e}^{-t}-\cos(a\,t))\,a}{1+a^2}$ **(5.4.1.7)**
Create a function so that you can quickly evaluate *yt* at any value of a. Use the $f := a \rightarrow y$ in the expression palette and use copy and past from the previous result to define this function.	$ytf := a \rightarrow \left(\frac{\sin(a\,t)+a\,(-\cos(a\,t)+\mathrm{e}^{-t})}{1+a^2}\right)$ $a \rightarrow \frac{\sin(a\,t)+a\,(-\cos(a\,t)+\mathrm{e}^{-t})}{1+a^2}$ **(5.4.1.8)**
Try a = 10. and plot using the menu and *Plotbuilder*.	$ytf(10)$ $\frac{\sin(10\,t)}{101}-\frac{10\cos(10\,t)}{101}+\frac{10\,\mathrm{e}^{-t}}{101}$ **(5.4.1.9)** $\rightarrow$

	
Wrap a *plot*(, $t = 0..10$) around the general expression for the response. Right click and choose *Explore*. You will then get a dialog box. Change the "to" value from 10 to 10.0. This will tell Maple that you are interested in values of a that are non-integer. This generates a new document with an *Exploration Assistant*. With this tool, you can simply slide a value of a and see the resulting plot. This is a general convenience tool that can be used for a wide range of Maple objects that depend on parameters.	$plot\left(\frac{\sin(a\,t) + a\,(-\cos(a\,t) + e^{-t})}{1 + a^2}, t = 0..10\right)$ 

▼ *Command Version* 

$G := \frac{1}{s+1}$

$$\frac{1}{s+1} \quad \textbf{(5.4.1.1.1)}$$

$rt := \sin(a\,t)$

$$\sin(a\,t) \quad \textbf{(5.4.1.1.2)}$$

$Rs := inttrans[laplace](rt, t, s)$

$$\frac{a}{s^2+a^2} \quad \textbf{(5.4.1.1.3)}$$

$Y := G\,Rs$

$$\frac{a}{(s+1)\,(s^2+a^2)} \quad \textbf{(5.4.1.1.4)}$$

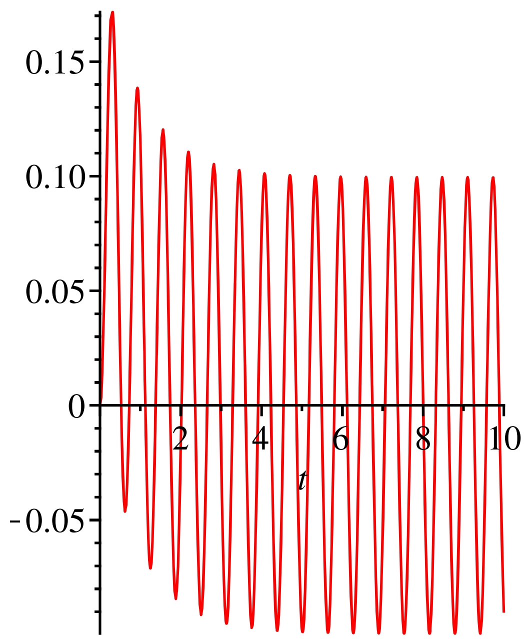
$yt := inttrans[invlaplace](Y, s, t)$

$$\frac{\sin(a\,t) + (\mathrm{e}^{-t} - \cos(a\,t))\,a}{1+a^2} \quad \textbf{(5.4.1.1.5)}$$

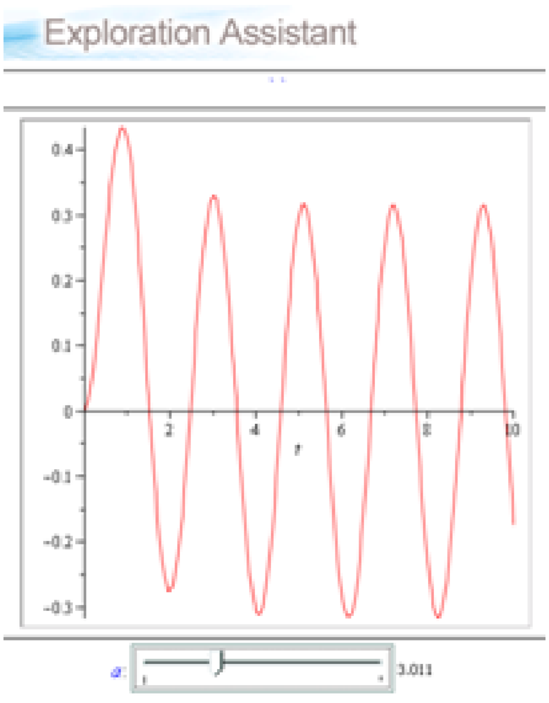
$ytf := unapply(yt, a)$

$$a \rightarrow \frac{\sin(a\,t) + (\mathrm{e}^{-t} - \cos(a\,t))\,a}{1+a^2} \quad \textbf{(5.4.1.1.6)}$$

$plot(ytf(10), t = 0..10)$



$Explore\left(plot\left(\frac{\sin(a\,t) + a\,(-\cos(a\,t) + \mathrm{e}^{-t})}{1+a^2}, t = 0..10\right)\right)$



Go back to Table of Contents

5.4.2 Bode plots using the DynamicSystems package

Load *DynamicSystems* package.

with(*DynamicSystems*)

[*AlgEquation*, *BodePlot*, *CharacteristicPolynomial*, *Chirp*, *Coefficients*, *ControllabilityMatrix*, *Controllable*, *DiffEquation*, *DiscretePlot*, *FrequencyResponse*, *GainMargin*, *Grammians*, *ImpulseResponse*, *ImpulseResponsePlot*, *IsSystem*, *MagnitudePlot*, *NewSystem*, *ObservabilityMatrix*, *Observable*, *PhaseMargin*, *PhasePlot*, *PrintSystem*, *Ramp*, *ResponsePlot*, *RootContourPlot*, *RootLocusPlot*, *RouthTable*, *SSModelReduction*, *SSTransformation*, *Simulate*, *Sinc*, *Sine*, *Square*, *StateSpace*, *Step*, *System*, *SystemOptions*, *ToDiscrete*, *TransferFunction*, *Triangle*, *Verify*, *ZeroPoleGain*, *ZeroPolePlot*] **(5.4.2.1)**

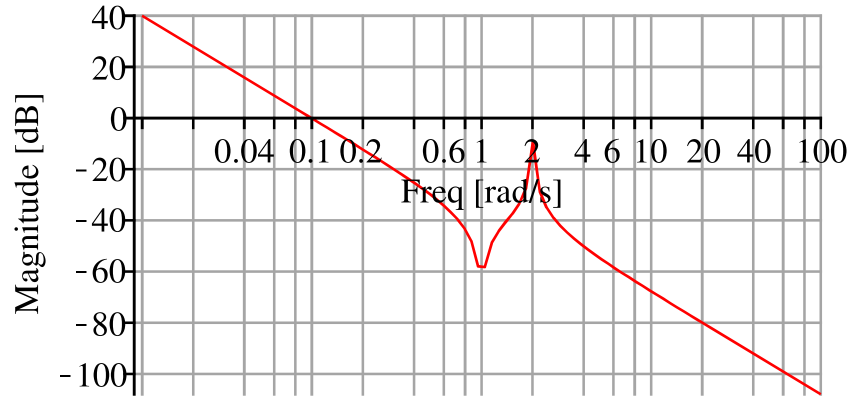
Define a transfer function as a system compatible with the *DynamicSystems* package.

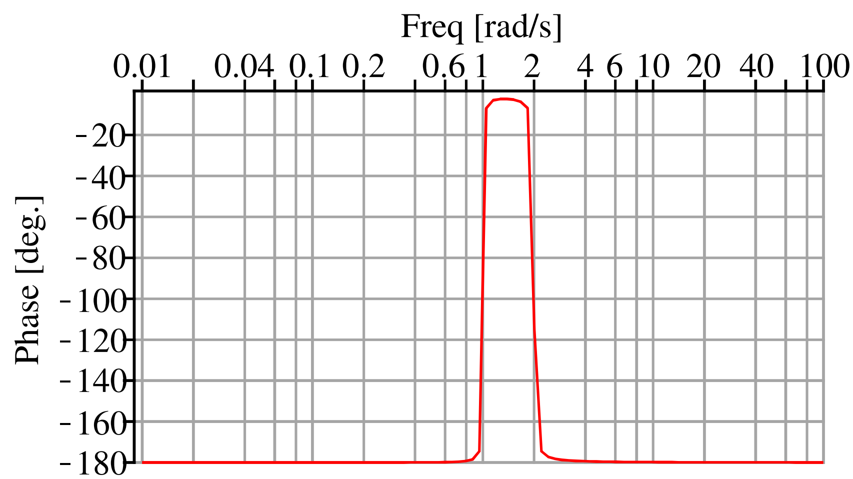
$$KGs := NewSystem\left(\frac{0.01\left(s^2+0.01\,s+1\right)}{s^2\left(\frac{s^2}{4}+\frac{0.02\,s}{2}+1\right)}\right)$$

Transfer Function
continuous
1 output(s); 1 input(s)
inputvariable = [$u1(s)$]
outputvariable = [$y1(s)$] **(5.4.2.2)**

Construct the Bode plots.

BodePlot(*KGs*)





Go back to Table of Contents

5.4.3 Manual construction of Bode plots for a lead compensator

This is an illustrative example showing how your can use fundamental principles to construct a Bode plot step by step. It exploits the fact that the Bode plot is a special complex plot of the function that results when $s = i\,\omega$ where ω is the frequency variable in radians.

Define a transfer function.

$TF := \dfrac{s+1}{.1\,s+1}$

$$\frac{s+1}{0.1\,s+1} \qquad \textbf{(5.4.3.1)}$$

Let $s = i\,\omega$

$TFomega := eval\left(TF, s = I\,\omega\right)$

$$\frac{\mathrm{I}\,\omega+1}{0.1\,\mathrm{I}\,\omega+1} \qquad \textbf{(5.4.3.2)}$$

Convert function to Hertz with $\omega = 2\,\pi\,f$ where f is now the new frequency variable in Hertz.

$TFf := eval\left(TFomega, \omega = 2\,\pi\,f\right)$

$$\frac{2\,\mathrm{I}\,\pi\,f+1}{0.2\,\mathrm{I}\,\pi\,f+1} \qquad \textbf{(5.4.3.3)}$$

.

Compute the magnitude of the function. The Maple command is *abs*. The command *evalc* tells Maple to apply special rules pertaining to complex numbers and functions to compute the magnitude and phase.

$mag := \text{abs}(TFf)$

$$\left|\frac{2\,\mathrm{I}\,\pi f+1}{0.2\,\mathrm{I}\,\pi f+1}\right| \quad (5.4.3.4)$$

$magc := evalc(mag)$

$$\sqrt{\left(\frac{1}{1+0.04\,\pi^2 f^2}+\frac{0.4\,\pi^2 f^2}{1+0.04\,\pi^2 f^2}\right)^2+\frac{3.24\,\pi^2 f^2}{\left(1+0.04\,\pi^2 f^2\right)^2}} \quad (5.4.3.5)$$

Repeat for the phase. The Maple command for phase is *argument.*

$phase := \text{argument}(TFf)$

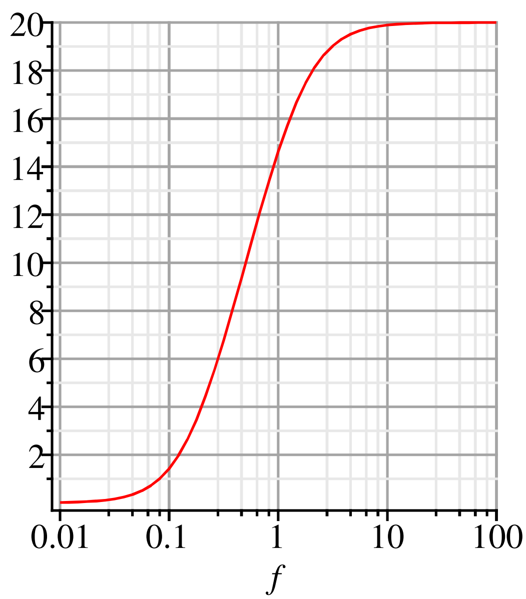
$$\text{argument}\left(\frac{2\,\mathrm{I}\,\pi f+1}{0.2\,\mathrm{I}\,\pi f+1}\right) \quad (5.4.3.6)$$

$phasec := evalc(phase)$

$$\arctan\left(\frac{1.8\,\pi f}{1+0.04\,\pi^2 f^2},\ \frac{1}{1+0.04\,\pi^2 f^2}+\frac{0.4\,\pi^2 f^2}{1+0.04\,\pi^2 f^2}\right) \quad (5.4.3.7)$$

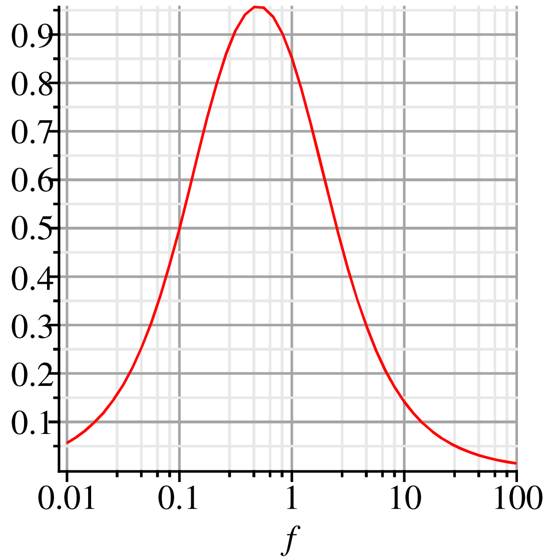
Use the *semilogplot* command from the *plots* package to plot on a logarithmic axis on the horizontal. To obtain dBs on the vertical, plot $20\log_{10} f$.

$plots[semilogplot](20\cdot\log[10](magc), f = .01..100, gridlines = true)$

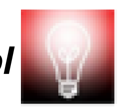


Repeat for phase without the dB conversion calculation of course.

$plots[semilogplot](phasec, f = .01..100, gridlines = true)$

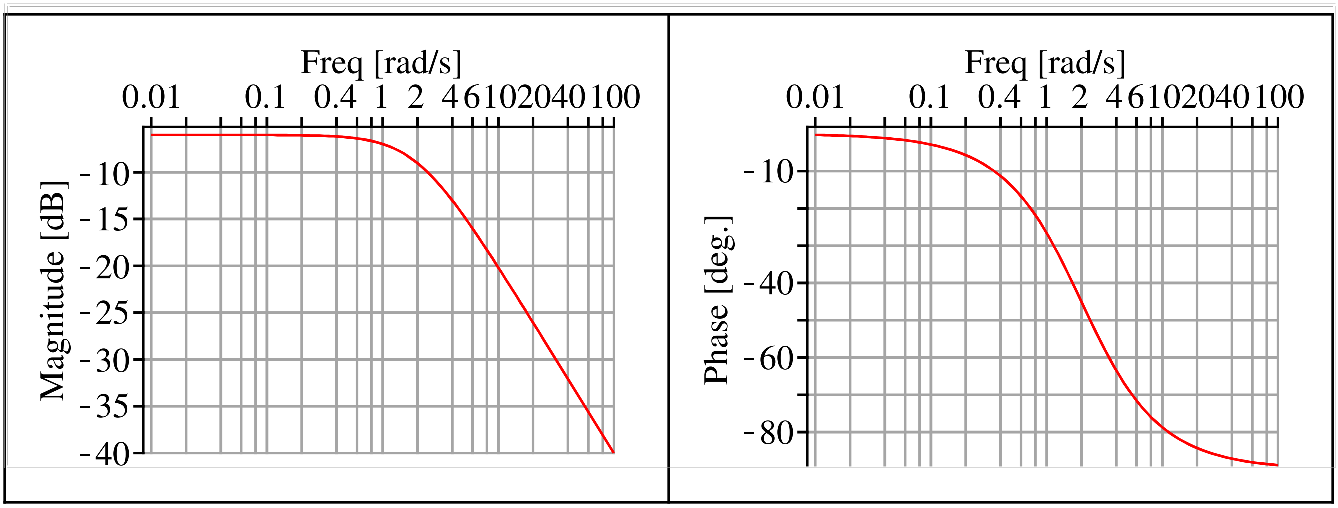


Go back to Table of Contents

5.4.4 Bode plot exploration tool 

This is an example of how Maple can be configured to provide easy to use interactive tools for complex processes. Directly enter or copy a transfer function into the math "container" and press the "Bode Plot" button below the plot region. If you want to see how this tool is made, open the "If you want to see the code" section.

$$\frac{1}{s+2}$$



Bode Plot

If you want to see the code

The following code is inserted into the "Bode Plot" button definition. To access, right-click on the button and choose *Component Properties→Edit.*

```
use DocumentTools in

  Do( tf=%transferfunction );
  tfs := DynamicSystems[NewSystem](tf);
  Do(%bodeplotregion = DynamicSystems[BodePlot](tfs));

end use;
```

Go back to Table of Contents

5.5 Frequency domain analysis: Nyquist plots

Unfortunately, Maple 12 has limited tools for Nyquist plots of frequency response. The following are two examples of certain Nyquist operations you can do with the current version.

5.5.1 Example: $\frac{1}{(s+1)^2}$

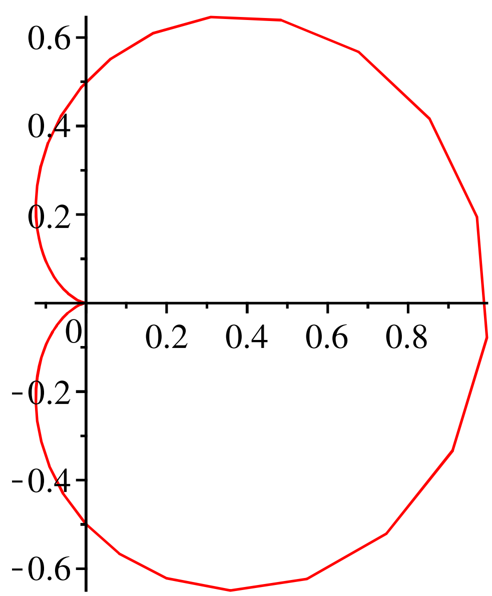
Define G as a proper Maple function of s. This will make it convenient to evaluate the function for different values of $i\,\omega$.

$G := s \rightarrow \left(\frac{1}{(s+1)^2}\right)$

$$s \rightarrow \frac{1}{(s+1)^2} \qquad \textbf{(5.5.1.1)}$$

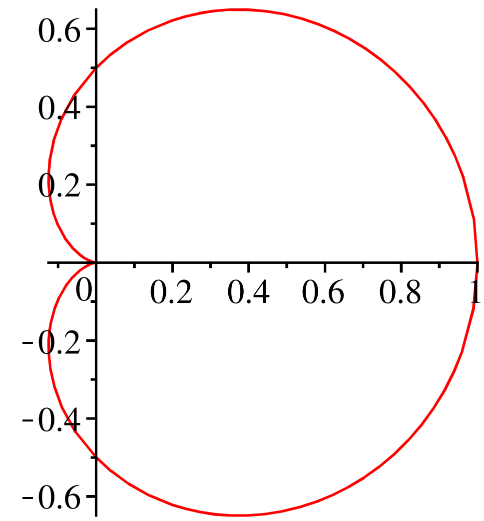
Use the *complexplot* command of the *plots* package that will reference the real values to the horizontal axis and imaginary values to the vertical axis.

$plots[complexplot]\left(G(I\,\omega),\ \omega = -100\,..\,100\right)$



Note the above plot is a bit course on the right. Increasing the number of plotting points smoothes this out.

$plots[complexplot](G(I\,\omega), \omega = -100..100, numpoints = 200)$



Go back to Table of Contents

5.5.2 Example: $\frac{1}{s(s+1)^2}$ 

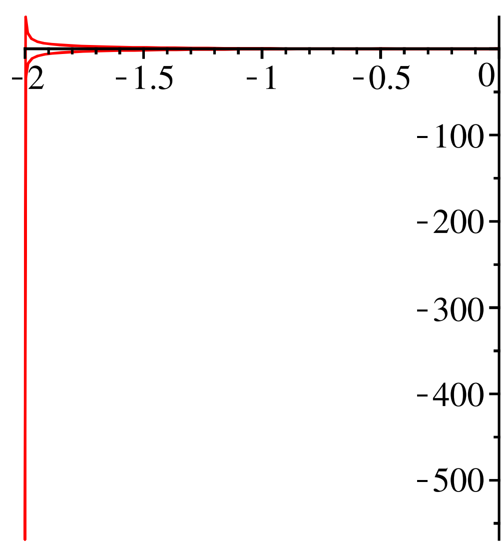
This is a more difficult example.

Define function and plot as before. The resulting plot is not very useful as this Nyquist plot tends to infinity.

$G := s \rightarrow \left(\frac{1}{s\,(s+1)^2}\right)$

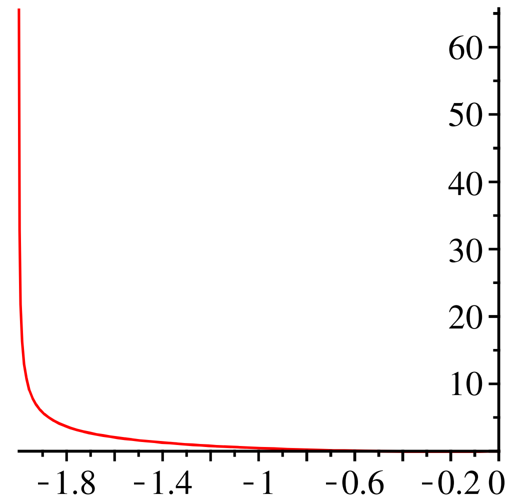
$$s \rightarrow \frac{1}{s\,(s+1)^2} \quad \textbf{(5.5.2.1)}$$

$plots[complexplot](G(I\,\omega), \omega = -100..100, numpoints = 200)$



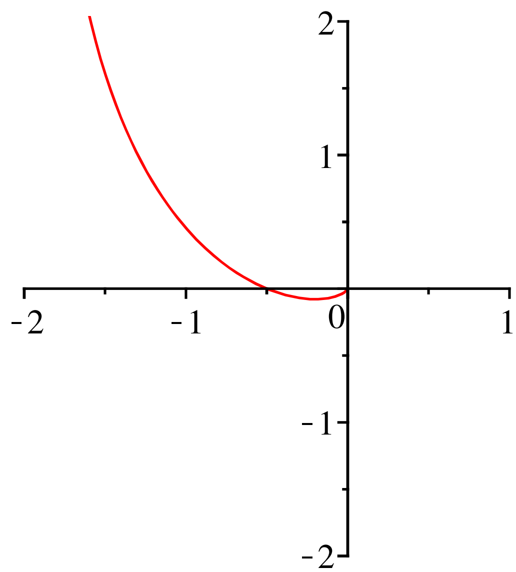
Try plotting only the negative values of frequency. Try $\omega = -100$ to 0. This time, the plot looks better but lacks the detail near the origin which is a very important region for Nyquist analysis.

plots[*complexplot*]($G(I\omega)$, ω =−100 .. 0, *numpoints* = 200)



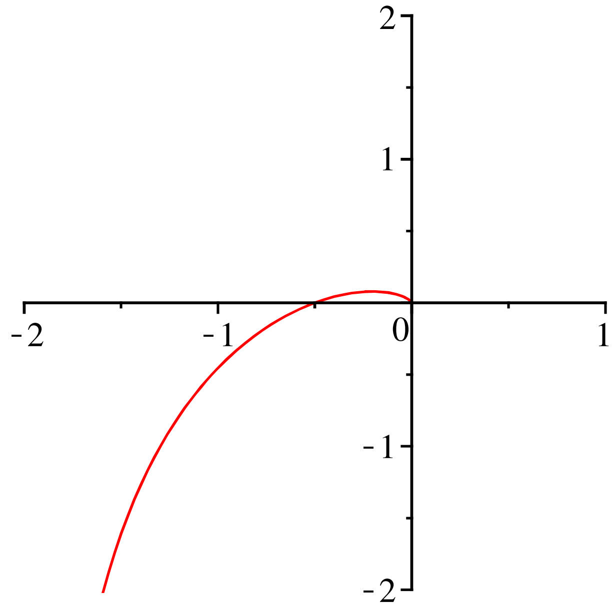
Now try restricting the view of the plot to around the origin. change the *view* option within the *complexplot* command. Now you get a good sense of the behavior near the origin.

plots[*complexplot*]($G(I\omega)$, ω =−100 .. 0, *numpoints* = 200, *view* = [−2 ..1,−2 ..2])

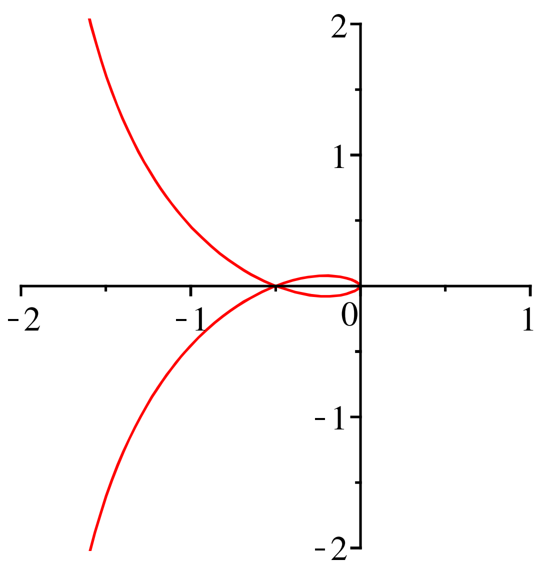


Repeat for positive frequencies. Combined, this and the previous plot constitute the the complete Nyquist plot for the region near the origin.

plots[*complexplot*]($G(I\omega)$, ω = 0 .. 100, *numpoints* = 200, *view* = [−2 ..1,−2 ..2])



 Note: One way to combine the two plots is simply to drag and drop one graph to the other. Click on the actual curve that you want to move then simply drag and drop. If you want to preserve the original plot, press [Ctl] on a Windows system when you drag. This is called a "drag copy". The following plot was created in such a way.

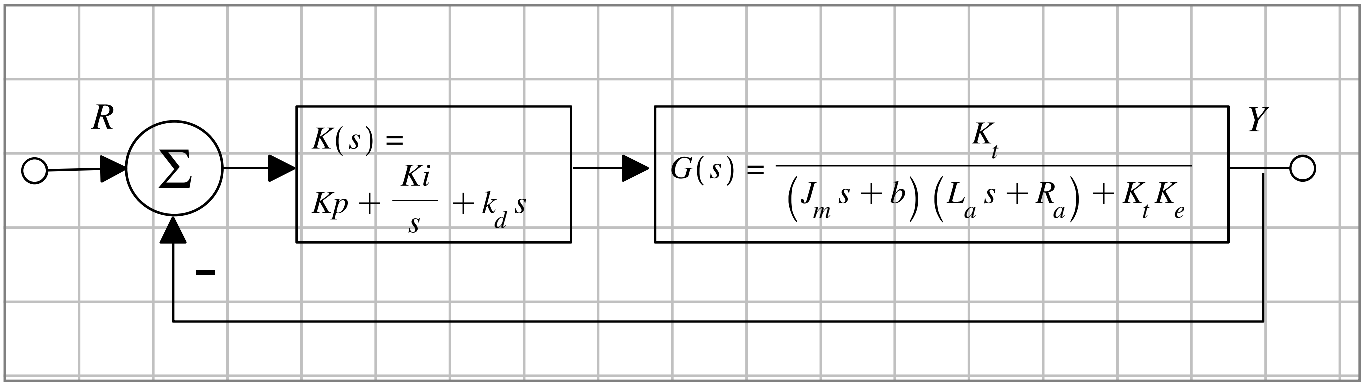


Go back to Table of Contents

6. PID Control

6.1 DC Motor definition

This example shows the development of the closed-loop transfer function for

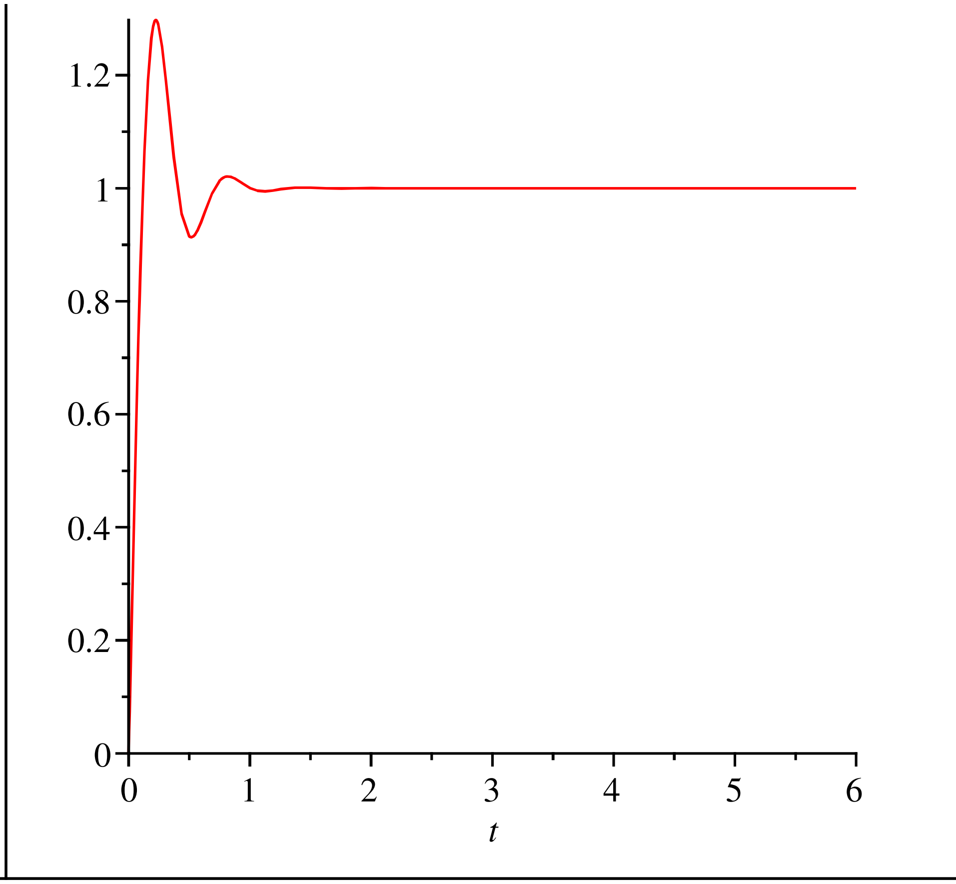


Operation	**Clickable** 
Define transfer function for both the motor plant ($G(s)$) and the controller $K(s)$ (named *PID* in this example). Assign to a name G and *PID* respectively. Assemble the closed loop transfer function.	$\frac{K_t}{(J_m s+b)(L_a s+R_a)+K_t K_e}$ $$\frac{K_t}{(J_m s+b)(L_a s+R_a)+K_t K_e} \quad (6.1.1)$$ $\xrightarrow{\text{assign to a name}}$ $$\frac{K_t}{(J_m s+b)(L_a s+R_a)+K_t K_e}$$ $k_p + \frac{k_i}{s} + k_d s$ $$k_p + \frac{k_i}{s} + k_d s \quad (6.1.2)$$ $\xrightarrow{\text{assign to a name}}$ $$k_p + \frac{k_i}{s} + k_d s$$ $\frac{G\,PID}{1+G\,PID}$ $$\frac{K_t\left(k_p+\frac{k_i}{s}+k_d s\right)}{\left((J_m s+b)(L_a s+R_a)+K_t K_e\right)\left(1+\frac{K_t\left(k_p+\frac{k_i}{s}+k_d s\right)}{(J_m s+b)(L_a s+R_a)+K_t K_e}\right)} \quad (6.1.3)$$

Given the transfer function, normalize the function (i.e. make it a simpler algebraic	$\xrightarrow{\text{normal}}$

fraction) and rearrange the denominator with respect to *s*. This will make the characteristic equation coefficients more readable.	$\frac{K_t (k_p s + k_i + k_d s^2)}{J_m s^3 L_a + J_m s^2 R_a + b L_a s^2 + s b R_a + s K_t K_e + K_t k_p s + K_t k_i + K_t k_d s^2}$ **(6.1.4)** collect w.r.t. s $=$ $\frac{K_t (k_p s + k_i + k_d s^2)}{J_m s^3 L_a + (J_m R_a + b L_a + K_t k_d) s^2 + (K_t K_e + K_t k_p + b R_a) s + K_t k_i}$ **(6.1.5)**
Use the $\left[f(x) \Big\vert_{x=a} \right]$ tool in the Expression Palette. Copy and paste the previous result (the transfer function), and specify the values of the various parameters.	$\left(\frac{K_t (k_p s + k_i + k_d s^2)}{J_m s^3 L_a + (J_m R_a + b L_a + K_t k_d) s^2 + (K_t K_e + K_t k_p + b R_a) s + K_t k_i}\right) \Bigg\vert (J_m = 1.13\,10^{-2}, b = 0.028, L_a = 10^{-1}, R_a = 0.45, K_t = 0.067, K_e = 0.067)$ $(0.067 (k_p s + k_i + k_d s^2)) / (0.001130000000 s^3 + (0.007885000000 + 0.067 k_d) s^2 + (0.017089 + 0.067 k_p) s + 0.067 k_i)$ **(6.1.6)** assign to a name $\longrightarrow$ $(0.067 (k_p s + k_i + k_d s^2)) / (0.001130000000 s^3 + (0.007885000000 + 0.067 k_d) s^2 + (0.017089 + 0.067 k_p) s + 0.067 k_i)$ **(6.1.7)**

Specify the input as a step function ($U(s) = \frac{1}{s}$) and multiply with the closed loop transfer function to obtain the output $Y(s)$.	$\frac{1}{s}\, TF$ $(0.067 (k_p s + k_i + k_d s^2)) / (s (0.001130000000 s^3 + (0.007885000000 + 0.067 k_d) s^2 + (0.017089 + 0.067 k_p) s + 0.067 k_i))$ **(6.1.8)**
Specify controller parameter values.	$\left(\frac{0.067 (3 s + 15 + 0.15 s^2)}{s (0.001130000000 s^3 + 0.01793500000 s^2 + 0.218089 s + 1.005)}\right) \Bigg\vert (k_p = 3, k_i = 15, k_d = 0.15)$ $\frac{0.067 (3 s + 15 + 0.15 s^2)}{s (0.001130000000 s^3 + 0.01793500000 s^2 + 0.218089 s + 1.005)}$ **(6.1.9)**
Compute the inverse Laplace transform using the right click menu, *Integral Transforms→Inverse Laplace*. Choose defaults for variables when dialog appears. With the right-click, *Plot Builder*, plot the time response function with *t* from 0 to 6.0.	inverse Laplace transform $\longrightarrow$ $-0.1186250615\, e^{-6.769420173\, t} + (-0.4406874693 + 0.1938949778\, I)\, e^{(-4.551130621 - 10.51994744\, I)\, t} - (0.4406874693 + 0.1938949778\, I)\, e^{(-4.551130621 + 10.51994744\, I)\, t} + 1$ **(6.1.10)** $\rightarrow$



Command Version

$$G := \frac{K_t}{(J_m s + b)(L_a s + R_a) + K_t K_e}$$

$$\frac{K_t}{(J_m s + b)(L_a s + R_a) + K_t K_e} \qquad \textbf{(6.1.1.1)}$$

$$PID := k_p + \frac{k_i}{s} + k_d s$$

$$k_p + \frac{k_i}{s} + k_d s \qquad \textbf{(6.1.1.2)}$$

$$TF := \frac{G\,PID}{1 + G\,PID}$$

$$\frac{K_t\left(k_p + \frac{k_i}{s} + k_d s\right)}{\left((J_m s + b)(L_a s + R_a) + K_t K_e\right)\left(1 + \frac{K_t\left(k_p + \frac{k_i}{s} + k_d s\right)}{(J_m s + b)(L_a s + R_a) + K_t K_e}\right)} \qquad \textbf{(6.1.1.3)}$$

$$TF := collect(\,normal(TF), s)$$

$$\frac{K_t\left(k_p s + k_i + k_d s^2\right)}{J_m s^3 L_a + \left(J_m R_a + b L_a + K_t k_d\right) s^2 + \left(K_t K_e + K_t k_p + b R_a\right) s + K_t k_i} \qquad \textbf{(6.1.1.4)}$$

$$TF := eval\left(TF, \left[J_m = 1.13\,10^{-2}, b = 0.028, L_a = 10^{-1}, R_a = 0.45, K_t = 0.067, K_e = 0.067\right]\right)$$

$$\frac{0.067\left(k_p s + k_i + k_d s^2\right)}{0.001130000000\,s^3 + \left(0.007885000000 + 0.067\,k_d\right) s^2 + \left(0.017089 + 0.067\,k_p\right) s + 0.067\,k_i} \qquad \textbf{(6.1.1.5)}$$

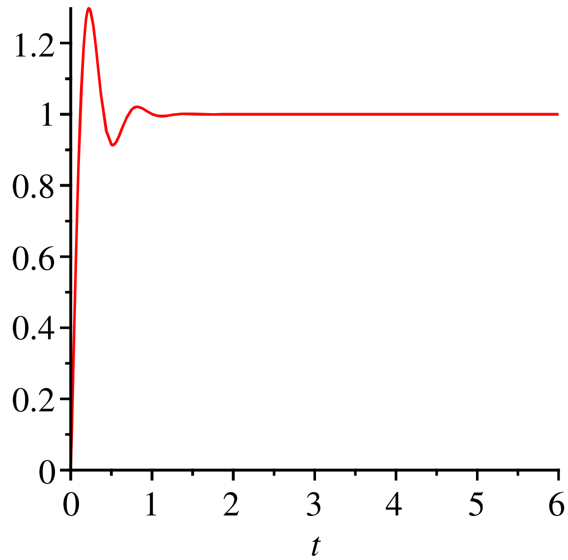
$$Ys := \frac{1}{s}\,TF$$

$$\frac{0.067\left(k_p s + k_i + k_d s^2\right)}{s\left(0.001130000000\,s^3 + \left(0.007885000000 + 0.067\,k_d\right)s^2 + \left(0.017089 + 0.067\,k_p\right)s + 0.067\,k_i\right)} \quad \textbf{(6.1.1.6)}$$

$$Ys := eval\left(Ys, \left[k_p = 3, k_i = 15, k_d = 0.15\right]\right)$$

$$\frac{0.067\,(3\,s + 15 + 0.15\,s^2)}{s\,(0.001130000000\,s^3 + 0.01793500000\,s^2 + 0.218089\,s + 1.005)} \quad \textbf{(6.1.1.7)}$$

$$plot(inttrans[invlaplace](Ys, s, t),\, t = 0\,..6)$$



[Go back to Table of Contents](#)

6.2 PID Exploration tool 

This is an example of using some of Maple's interactive user interface building tools (*Components* palette) to create a user-friendly tool that removes the need to know any commands or menu options.

For this example:

1. Compute the closed loop transfer function as in the previous example
2. Copy and paste the transfer function into the first region
3. Specify numerical values for the controller parameters
4. Press the [Response plot] button to compute the time response expression and the response plot

If you want to see the details of this example, look at the subsection "*If you want to see the code*" following this tool.

Copy transfer function expression into here

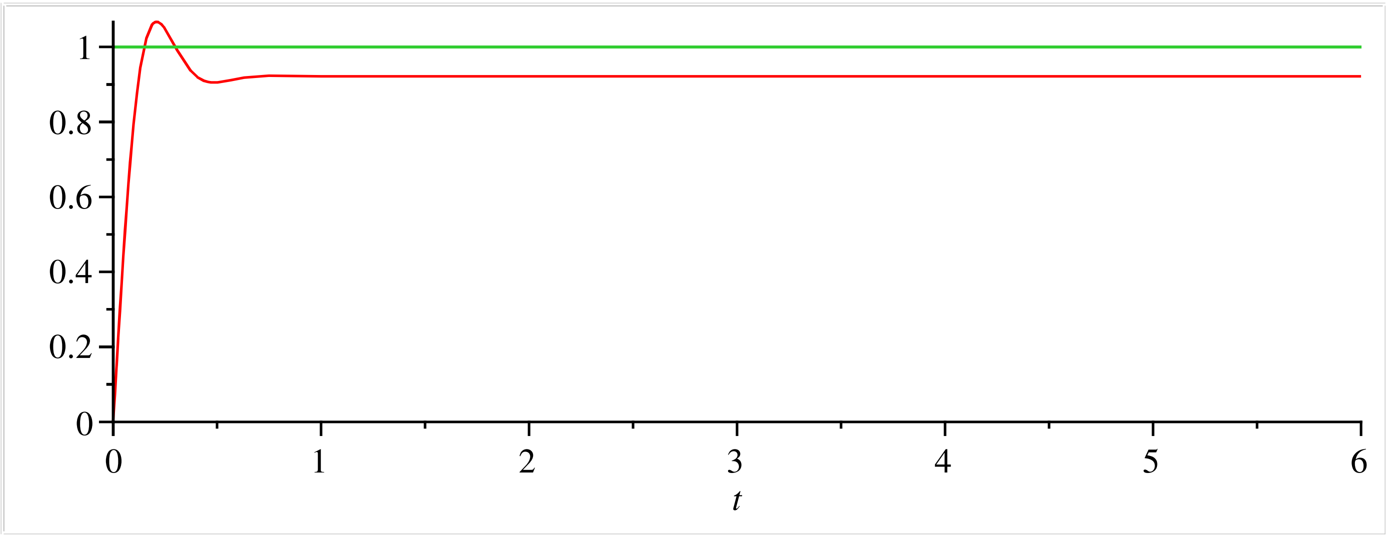
$$\frac{0.067\left(k_p s + k_i + k_d s^2\right)}{\left(0.001130000000 s^3 + \left(0.007885000000 + 0.067\, k_d\right) s^2 + \left(0.017089 + 0.067\, k_p\right) s + 0.067\, k_i\right) s}$$

Time response expression

$$0.9216 - 0.9218\, \mathrm{e}^{-7.936\, t} \cos(11.40\, t) + 0.1386\, \mathrm{e}^{-7.936\, t} \sin(11.40\, t)$$

Enter controller parameters (do not leave any parameter blank. Use 0.0 value instead)

k_p 3 k_i 0.0 k_d 0.15 Response plot



[Go back to Table of Contents](#)

If you want to see the code 

The following code is inserted into the "Response Plot" button definition. To access, right-click on the button and choose *Component Properties→Edit.*

```
use DocumentTools in

  # get PID values
  Do(k[p] = %kp);
  Do(k[i] = %ki);
  Do(k[d] = %kd);

  # final transfer function
```

```
  rs := 1/s;
  Do(tf = %transferfunction);
  ys := rs * tf;

  # inverse Laplace transform
  yt := evalf(simplify(inttrans[invlaplace](tf,s,t)),4);

  # put results in the expression and plot containers.
  Do(%responseexpression = yt);
  Do(%responseplot = plot([yt,Heaviside(t)],t=0..6));

end use;
```

To access any other definitions of components, simply right-click on the region or object and look up the properties. You will notice that the button is the only object that actually has any "smarts" all other objects simply take in or display information.

7. State Space Representation

7.1 Obtaining state space matrices with DynamicSystems package

Convert the transfer function $\frac{(s+2)}{s^2+7s+12}$ and assemble the state space equation $\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,u(t)$.

Load the *DynamicSystems* package.

with(*DynamicSystems*)

[*AlgEquation*, *BodePlot*, *CharacteristicPolynomial*, *Chirp*, *Coefficients*, *ControllabilityMatrix*, *Controllable*, *DiffEquation*, *DiscretePlot*, *FrequencyResponse*, *GainMargin*, *Grammians*, *ImpulseResponse*, *ImpulseResponsePlot*, *IsSystem*, *MagnitudePlot*, *NewSystem*, *ObservabilityMatrix*, *Observable*, *PhaseMargin*, *PhasePlot*, *PrintSystem*, *Ramp*, *ResponsePlot*, *RootContourPlot*, *RootLocusPlot*, *RouthTable*, *SSModelReduction*, *SSTransformation*, *Simulate*, *Sinc*, *Sine*, *Square*, *StateSpace*, *Step*, *System*, *SystemOptions*, *ToDiscrete*, *TransferFunction*, *Triangle*, *Verify*, *ZeroPoleGain*, *ZeroPolePlot*] **(7.1.1)**

Convert the transfer function to state space form. To actually see the various matrices, use the *PrintSystem* command. Store the result in the variable *SS*. Note the labels for the 4 state space matrices **A**, **B**, **C**, **D**. You will need these for the next steps.

$SS := StateSpace\left(\frac{(s+2)}{s^2+7s+12}\right)$

State Space
continuous
1 output(s); 1 input(s); 2 state(s)
inputvariable = [$u1(t)$]
outputvariable = [$y1(t)$]
statevariable = [$x1(t)$, $x2(t)$] **(7.1.2)**

PrintSystem(*SS*)

$$\begin{array}{c}\textbf{State Space}\\ \text{continuous}\\ \text{1 output(s); 1 input(s); 2 state(s)}\\ \text{inputvariable} = [u1(t)]\\ \text{outputvariable} = [y1(t)]\\ \text{statevariable} = [x1(t), x2(t)]\\ \text{a} = \begin{bmatrix} 0 & 1 \\ -12 & -7 \end{bmatrix}\\ \text{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\\ \text{c} = \begin{bmatrix} 2 & 1 \end{bmatrix}\\ \text{d} = \begin{bmatrix} 0 \end{bmatrix}\end{array} \quad (7.1.3)$$

Define the two vectors that you need for the general equation $\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,u(t)$.To enter the variable $(\dot{x}_1)$, make sure you are in math mode, then type [x] [_] (underscore) [1] [→] (right cursor). You should now see x_1. Now type, [shift][ctl] (hold both) ["]. The cursor will move to the top of the x. Now type [.] (period) for the dot. Finish with [→].

$$xdotvec := \begin{bmatrix} \frac{d}{dt} x_1(t) \\ \frac{d}{dt} x_2(t) \end{bmatrix}$$

$$\begin{bmatrix} \frac{d}{dt} x_1(t) \\ \frac{d}{dt} x_2(t) \end{bmatrix} \quad (7.1.4)$$

$$xvec := \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (7.1.5)$$

To get and use the matrix definitions:

$stateeqs := xdotvec = SS[a].xvec + SS[b].u(t)$

$$\begin{bmatrix} \frac{d}{dt} x_1(t) \\ \frac{d}{dt} x_2(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} . u(t) + \begin{bmatrix} x_2(t) \\ -12\,x_1(t) - 7\,x_2(t) \end{bmatrix} \quad (7.1.6)$$

Go back to Table of Contents

▼ 7.2 Controllability and observability with the DynamicSystems package

Load *DynamicSystems* package.

$with(DynamicSystems)$

[*AlgEquation*, *BodePlot*, *CharacteristicPolynomial*, *Chirp*, *Coefficients*, *ControllabilityMatrix*, *Controllable*, *DiffEquation*, *DiscretePlot*, *FrequencyResponse*, *GainMargin*, *Grammians*, *ImpulseResponse*, *ImpulseResponsePlot*, *IsSystem*, *MagnitudePlot*, *NewSystem*, *ObservabilityMatrix*, *Observable*, *PhaseMargin*, *PhasePlot*, *PrintSystem*, *Ramp*, *ResponsePlot*, *RootContourPlot*, *RootLocusPlot*, *RouthTable*, *SSModelReduction*, *SSTransformation*, *Simulate*, *Sinc*, *Sine*, *Square*, *StateSpace*, *Step*, *System*, *SystemOptions*, *ToDiscrete*, *TransferFunction*, *Triangle*, *Verify*, *ZeroPoleGain*, *ZeroPolePlot*] **(7.2.1)**

Define a transfer function of a system. Convert to state space form using the *StateSpace* command.

$$sys := StateSpace\left(\frac{1}{s^2+s+10}\right)$$

State Space
continuous
1 output(s); 1 input(s); 2 state(s)
inputvariable = [$u1(t)$]
outputvariable = [$y1(t)$]
statevariable = [$x1(t)$, $x2(t)$] **(7.2.2)**

Generate the required matrices.

$ControllabilityMatrix(sys)$

$$\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \quad \textbf{(7.2.3)}$$

$ObservabilityMatrix(sys)$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \textbf{(7.2.4)}$$

Query to see if the system is controllable or observable

$Controllable(sys)$

true **(7.2.5)**

$Observable(sys)$

true **(7.2.6)**

For this example, use a matrix definition of the state space model.

$$sysm := StateSpace\left(\begin{bmatrix} -3 & 1 & 0 \\ -5 & 0 & 1 \\ -3 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 \end{bmatrix}\right)$$

State Space
continuous
1 output(s); 1 input(s); 3 state(s)
inputvariable = [$u1(t)$]
outputvariable = [$y1(t)$]
statevariable = [$x1(t)$, $x2(t)$, $x3(t)$] **(7.2.7)**

$ControllabilityMatrix(sysm)$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & -3 & 3 \end{bmatrix} \tag{7.2.8}$$

Controllable (*sysm*)

$$\mathit{false} \tag{7.2.9}$$

[Go back to Table of Contents]